The Wits Maths Connect Secondary Project (WMCS) 2010 – 2019:
A research-linked professional development project and its impact

Asian Centre for Mathematics Education

ECNU – Shanghai – 30 March 2019

Professor Jill Adler SARChI Chair of Mathematics Education

University of the Witwatersrand

jill.adler@wits.ac.za

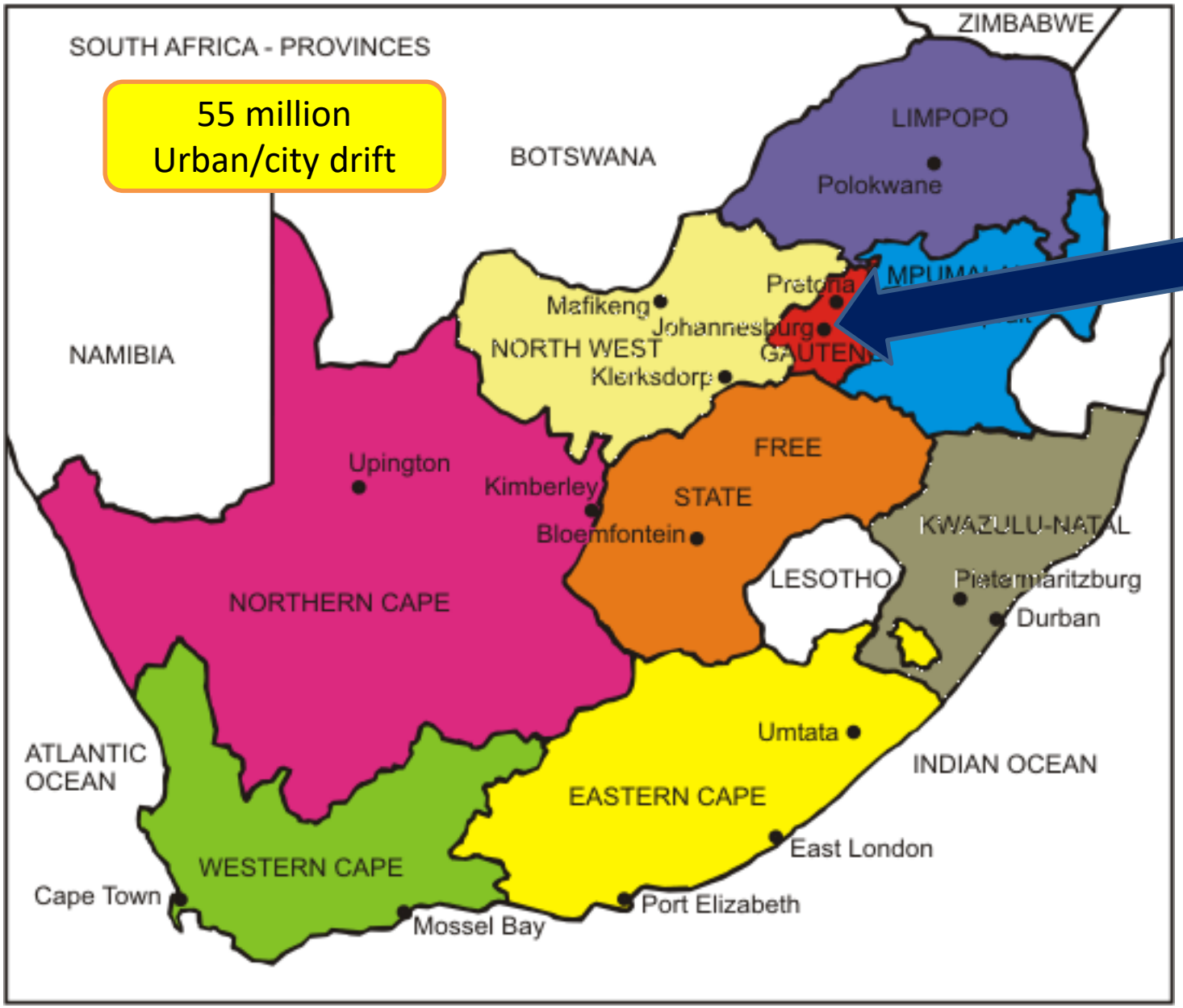
Overview of talk

- The South African education context – some comments

- The WMCS ***Mathematics for Teaching*** course
  - The ***model***
  - Its ***guiding framework***
  - ***Researching*** its impact on ***teacher knowledge and learner attainment***

- WMCS Complementary ***Lesson Study***
  - The ***model***
  - ***Research on lesson cycles -*** the significance of the ***guiding framework and knowledgeable other***

THE SOUTH AFRICAN EDUCATION CONTEXT


SOUTH AFRICA - PROVINCES
55 million
Urban/city drift
ZIMBABWE
BOTSWANA
LIMPOPO
Polokwane
Pretoria
Mafikeng
Johannesburg
NORTH WEST
GAUTENG
Klerksdorp
NAMIBIA
FREE
STATE
Upington
Kimberley
Bloemfontein
KWAZULU-NATAL
LESOTHO
Pietermaritzburg
Durban
NORTHERN CAPE
Umtata
ATLANTIC
OCEAN
INDIAN OCEAN
EASTERN CAPE
East London
WESTERN CAPE
Cape Town
Port Elizabeth
Mossel Bay

The WMCS Project schools

- No fee and low fee secondary schools in and around Johannesburg (80+)
- Most are poorly resourced with large classes
- English is language of instruction
- English is not main language of teachers or learners
- Many teachers in lower grades are not qualified to teach secondary mathematics
- Most learners perform very poorly in Mathematics and very few can access tertiary education that requires mathematics

The model

- Developed and refined over time in the first years of the project
- Through work with teachers, learner assessments and classroom observation
- “Grounded” in realities of conditions and practices in the schools and mathematics classrooms

Transition Maths 1 course

The major goal of this course is for participants to benefit and succeed – both in learning mathematics and in working on teaching. We aim to create a **professional learning environment** where teachers are able to work together to:

- **deepen and extend their knowledge of key topics in the school curriculum**
- **teach mathematics effectively at Grade 9 and 10 level,** and continue to other higher levels

We believe that this will support ….

- **more learners to succeed in Grade 9 and continue studying mathematics in Grades 10, 11 and 12**

Course content

- 8 x 2-day units over 1 year (16 days)
- Mathematics (75%)
  - Algebra (4.5 days)
  - Function (4.5)
  - Geometry (3.5)
  - Trigonometry (3.5)
- Teaching (25%)
  - Examples, tasks, representations
  - Explanations and justifications
  - Learner participation
- Independent work between course days

Our approach to maths in PD

- Revisiting known mathematics
  - Build on, strengthen and extend teachers' existing knowledge
  - Challenge and enable teachers to think and communicate mathematically
  - Make connections between different representations, different sections of the curriculum

- Learning new mathematics
  - "may as well be new"
  - Beyond the grade-level currently taught
  - Related to school curriculum
  - Extends beyond curriculum

Revisiting inequalities

Is the statement on the card ***true*** or ***false***?

1 $3 < 10$	2 $-3 < -10$
3 $10 \leq 10$	4 $5 > -5000$
5 $9 - 4 > 5$	6

Attention to reasoning and communicating mathematically

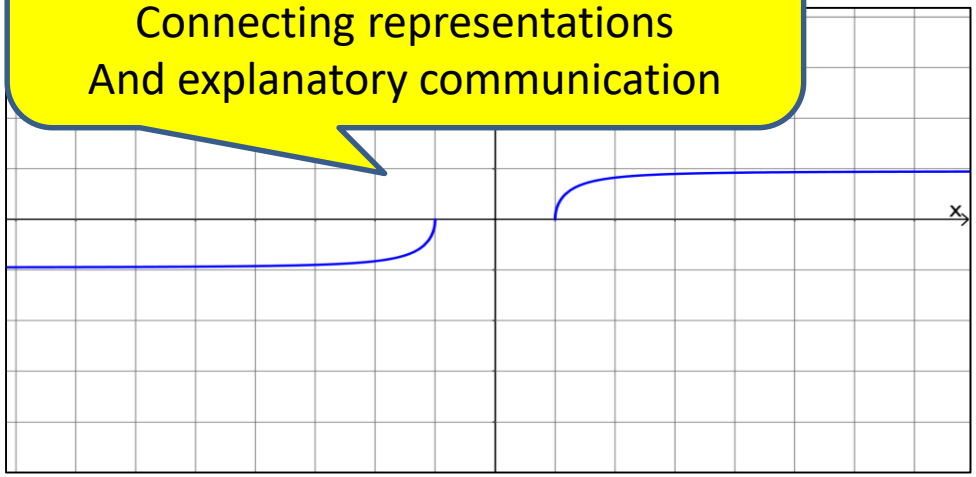
Is the statement ***always true***, ***sometimes true*** or ***never true***?

7 $x^2 > 0$	8 $-x < 0$
9 $(m-4)^2 > 0$	10 $(p+2)^2 > 2$

Attention to examples and variation amidst invariance

New content: functions

Attention to example(s)
Tasks
Connecting representations
And explanatory communication



Talker describes graph

Listener draws graph based on talker's descriptions

What words/phrases did the talker use that were helpful/not helpful?

The equation of the function is:

$$f(x) = \frac{\sqrt{x^2-1}}{x}$$

1) Why is there no graph from $x = -1$ to $x = 1$?
2) What is the domain of f?
3) Does the graph touch $y = 1$ and $y = -1$?
4) Why does the graph lie in quadrants I and III only?
5) Make one change to the equation so that the graph appears in (a) II and IV only, (b) I and II only

Our approach to mathematics teaching

We seek to strengthen current teaching practices through explicit study of mathematics teaching using research-informed resources

- ***Explicit*** study of mathematics teaching

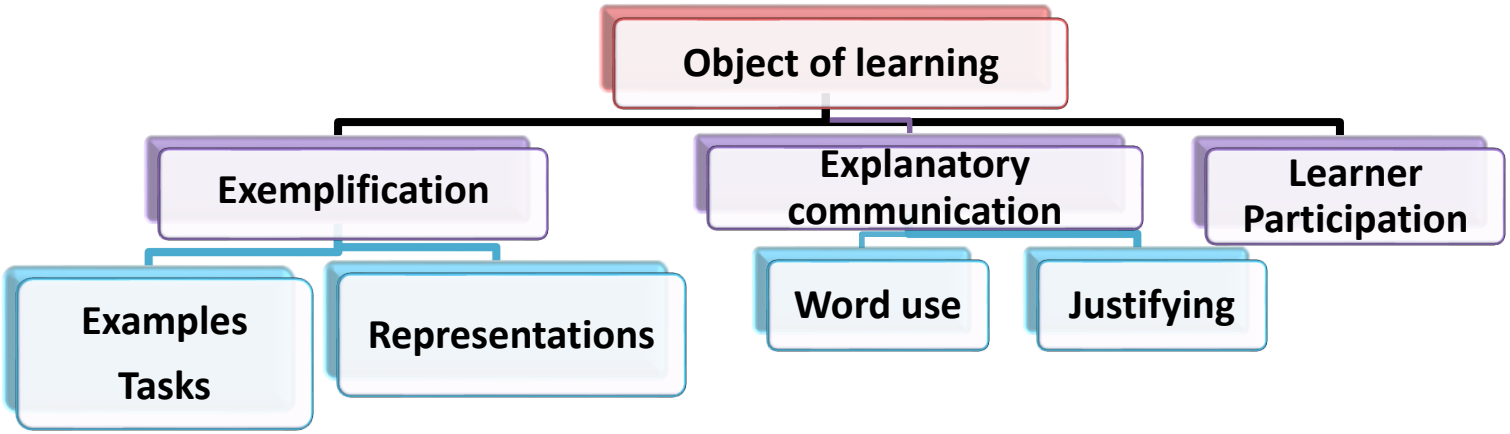
- ***Current*** *teaching practices*
  - We work close to existing practice
  - We acknowledge the conditions of teachers' work and work towards cultural resonance

- ***Strengthen*** *practices* = shift five aspects of teachers' practice
  - Exemplification – towards *deliberate selection of example sets*
  - Explanatory talk – towards *mediated* explanations
  - Learner activity – towards *active* participation
  - The mathematical message - from *fragmented* to *coherent and connected*

The framework

Mathematical discourse in instruction (MDI):

A socio-cultural framework for describing and studying/working on mathematics teaching



Mediational means

Cultural tools "resources"

Grounded in teaching practices

Mediation towards scientific concepts

Mathematics as network of connected "concepts"

Building generality and appreciating structure

Impact?

Qualitative studies

Quantitative studies

Learning Gains 2013 overview

Goal

Linking teachers' participation in PD with learner achievement over 1 academic year

Quasi-experimental

Pre-test: Feb 2013

Post-test: Oct 2013

Sample

Grade 10 learners in 5 schools

	Learners	Teachers
TM1	392	10
Control	217	7
	609	**17**

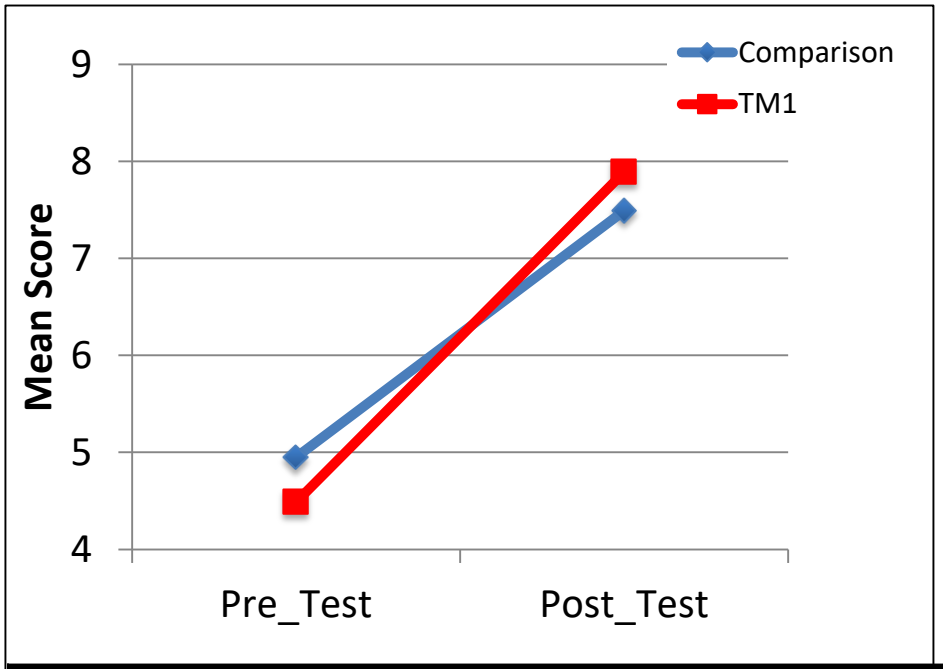
Test

- Typical curriculum items for Grade 10
- Algebra, functions, Euclidean geometry
- Rasch analysis – fit for purpose

Coding

- Test responses marked as *correct*, *incorrect* or *missing*
- Learning gains = No. of completely correct responses

Learning Gains 2013



Learners taught by teachers who had done the TM1 course out-performed learners in the same schools taught by teachers who had not done the TM1 course.

		Pre-test		Post-test		Gain	
	N	Mean	SD	Mean	SD	Mean	SD
TM1	392	4.49	3.35	7.89	5.32	3.40	4.03
Comparison	217	4.95	3.63	7.49	5.01	2.54	3.28

The practical significance of the results

Gain	Pooled SD	Effect size (d)	Equivalent progress
0.80	3.78	0.21	3 months

Higgins et al (2012)
Teaching and Learning Toolkit
Education Endowment Trust

Pournara, C., Hodgen, J, Adler, J. & Pillay, V. (2015). Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in Mathematics? *South African Journal of Education*, 35 (3).

Limitations

- Low scores
- Small gains
- High variation in scores
- **Indicative results**
- **Evidence of promise for TM1 as a PD intervention**

Learning Gains II study 2016-2019

Purpose

- More rigorous study to investigate impact of TM1 on learner attainment
- Build on results and learnings of Learning Gains study 2013
- Grade 9 & 10 learners

Design

- Quasi-experimental
- Larger sample of teachers, learners and schools
- Matched control group (based on school and teacher profile)
- New test instrument
- Pre-post design
  - 1st year after completing TM1
  - 2nd year after completing TM1
- Measure **learning gains**

  = Change in test score = No. of completely correct responses

New test instrument

Test content

- Number, algebra, function
- Grade 7, 8, 9 content
- 45 items

Piloting of test items

- Feb, May, Sept 2016
- Grade 9 and 10 learners in 3 schools
- Task-based interviews with Grade 10s

Tension in test design

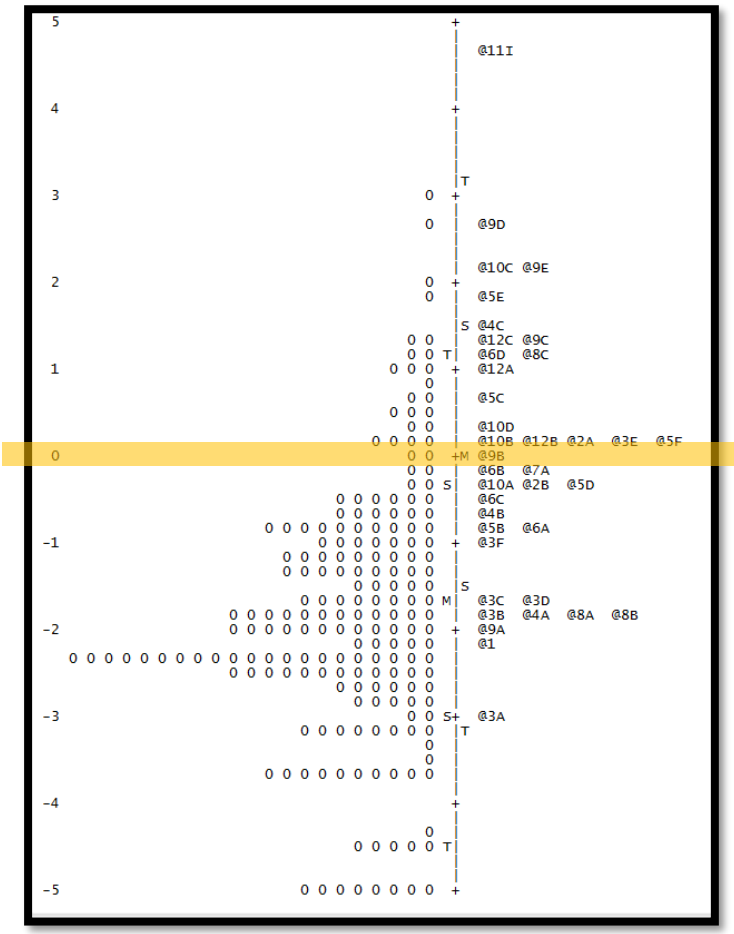
Test that will show learning gains

to “prove that PD worked”

vs

Test of grade-level competence

with “guarantee” of flooring effect



Learning Gains 2018 sample

	Grade 9			Grade 10		
Teacher group	No. of teachers	No. of schools	No. of learners	No. of teachers	No. of schools	No. of learners
TM1 2016	12	11	478	9	8	375
TM1 2017	12	10	544	6	6	190
Comparison	23	15	886	24	16	645
TOTAL	**47**	**36**	**1908**	**39**	**30**	**1210**

Test

- Content: Number, algebra, function
- 45 items
- Grades: 7, 8, 9
- Developed by project team, piloted in 2016
- Administered in Feb/March and Sept/Oct 2018
- Approx 1 hour

Coding

- Responses coded as

  correct, *incorrect* or *missing*
- Learning gains = No. of completely correct responses

Did TM1 impact teachers' knowledge?

		All teachers completing TM1 course				TM1 teachers in Learning Gains study			
		N	Ave mk	SD	t-test	N	Ave mk	SD	t-test
2016	Pre-test	40	61.58	15.79	t = 8.73 df = 39 $p < 0.001$	16	60.42	13.61	t = 5.17 df = 15 $p < 0.001$
	Final test	40	75.91	18.25		16	76.43	15.08	

Repeated sample t-tests show that the increases are statistically significant

TM1 had a significant impact on teachers' MfT
for the 2016 and 2017 cohorts

TM1 had a significant impact on teachers' MfT
for teachers in the LG study

Did TM1 impact learner attainment?

		Pre-test		Post-test		
Cohort	N	Ave mark	SD	Ave mark	SD	Change in Ave mark
Combined TM1 group	1587	8.54	6.50	11.90	8.44	3.36
Comparison group	1531	10.81	8.02	13.58	9.27	2.78

A repeated measures ANOVA shows that the **gains are statistically significant**

F	*df*	*p*
9.321	1	< 0.001

Effect size: 0.2

Gains small

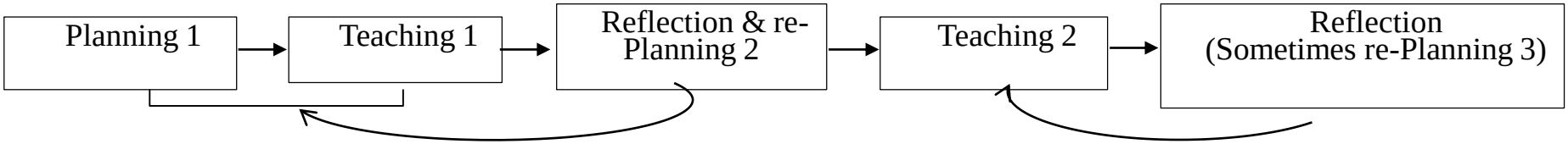
The guiding framework

and our

WMCS Lesson Study

WMCS "Lesson Study" model

A group of teachers, work together with a *knowledgeable other (more experienced teacher, researcher)* plan, teach, reflect on a lesson and then repeat



After school, once a week, one cycle a term at most (3 weeks)

A key concept; Particular learner difficulties; Teaching difficulties

A framework for working on teaching

LS in three clusters of schools, in 2016

A specific question in the study was on the evolution of the example set and in particular:

- What changes occur in the example set across the lesson plans over a cycle?
- How do these changes evolve?

WMCS Mathematics Teaching Framework

Structuring resource guiding planning and reflection

Lesson goal		
Exemplification Examples, tasks and representations	**Learner Participation** Doing maths and talking maths	**Explanatory communication** Word use and justifications
Building generality Structure Variation amidst invariance	Who is? Doing maths Talking maths	Informal – formal Mathematical substantiations Principles

What I have learned from these lesson studies, it is not only about my teaching and the strategies I use, but it is about … the mathematics that is behind each and every topic …

The reflection has helped a lot. We can reflect on what language are the learners using … **And most of all looking at the examples, that is one where we have grown a lot, what examples we use and why we use such examples**.

A problem our teachers identified

Simplify the following expressions

a) $2p - (4 + p) =$

b) $2p\ (-4 + p) =$

c) $(2 + p) - (4 + p) =$

Grade 10 "simplifying algebraic expressions with brackets in different positions"

Simplify: $2p - (4 + p) =$

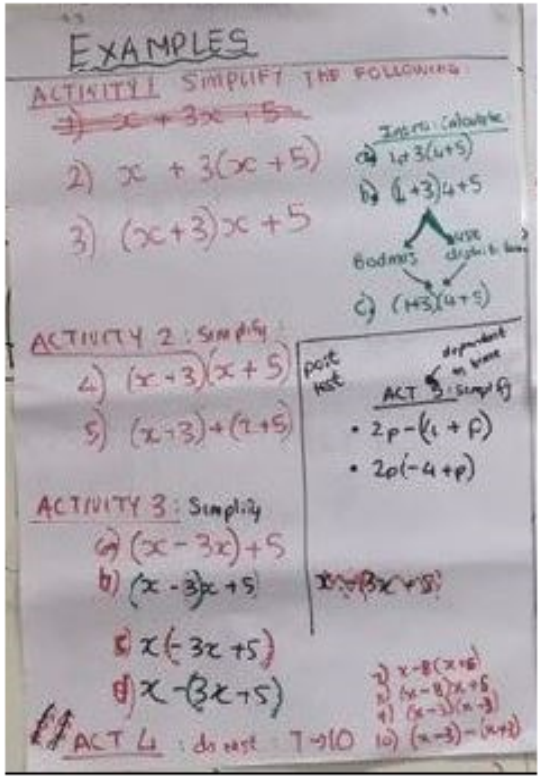
$-2p$; $8p - 2p^2$

"Do what is in the brackets first"

"Brackets means multiply"

- Reliance on "short cuts"?
- Rules without rationales?
- Overgeneralising?

Planned example set for lesson 1



EXAMPLES

Activity 1 Simplify the following

~~1) $x + 3x + 5$~~

2) $x + 3(x + 5)$

3) $(x + 3)x + 5$

Intro: Calculate

a) $4 + 3(4 + 5) =$

b) $(1 + 3)4 + 5 =$

Bodmas — use distributive law

c) $(1 + 3)(4 + 5)$

Activity 2: Simplify

4) $(x + 3)(x + 5)$

5) $(x + 3) + (x + 5)$

post test — dependent on time

ACT 5: Simplify

- $2p - (4 + p)$
- $2p(-4 + p)$

Activity 3: Simplify

a) $(x - 3x) + 5$

b) $(x - 3)x + 5$

c) $x(-3x + 5)$

d) $x - (3x + 5)$

ACT 4: do rest 7→10

7) $x - 8(x + 6)$

8) $(x - 8)x + 6$

9) $(x - 3)(x + 3)$

10) $(x - 3) - (x + 3)$

Introduction through numerical expressions

Pre- & post tests

wits maths connect
supporting secondary maths

Joint plan First Lesson

Lesson goal: Learners can simplify expressions with brackets when these are in different positions.

Calculate (simplify)

a) 4 + 3 (4 + 5) =

b) (4 + 3)4 + 5 =

c) (4 + 3)(4 + 5) =

Activity 3: Simplify

1. (x-3x)+5=
2. (x-3)x+5=
3. x(-3x+5)=
4. x-(3x+5)=

Learners calculate/ simplify on their own

Compare and discuss answers

What is the same and what is different in each of a, b and c?

And 1, 2, 3 & 4

(Same numbers/terms, brackets in different positions)

How does that change your calculations/ simplifications?

Plan 1	Plan 2	Plan 3
Pre-test assessment	Pre-test assessment	Pre-test assessment
Introduction: Calculate the following: a) $4+3(4+5)=$ b) $(4+3)4+5=$ c) $(4+3)(4+5)$	Introduction: simplify the following expressions: a) $4+3(4+5)=$ b) $(4+3)4+5=$ c) $(4+3)(4+5)=$	Introduction: simplify the following expressions: a) $4+3(4+5)=$ b) $(4+3)4+5=$ c) $(4+3)(4+5)=$ d) $(4+3)-(4+5)=$ Insert brackets in the left side to result in the given sum e) $4-3+5=-4$ f) $4+3-5=-35$
Activity 1: Simplify the following $x+3(x+5)=$ $4+3(4+5)=$ $(x+3)x+5=$ $(4+3)4+5=$	Activity 1: Simplify the following $x+3(x+5)=$ $(x+3)x+5=$	Activity 1: Simplify the following 1. $x+3(x+5)=$ 2. $(x+3)x+5=$ 3. $x-3(x+5)=$ 4. $-(x+3)x-5=$
Activity 2: Simplify $(x+3)(x+5)=$ $(x+3)+(x+5)=$	Activity 2: Simplify $(x+3)(x+5)=$ $(x+3)-(x+5)=$	Activity 2: Simplify 5. $(x+3)(x+5)=$ 6. $(x+3)-(x+5)=$ 7. $(x+3)(-x+5)=$
Activity 3: Simplify a) $(x-3x)+5=$ b) $(x-3)x+5=$ c) $x(-3x+5)=$ d) $x-(3x+5)$	Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal a) $x-3x+5=-3x^2+5x$ b) $x-3x+5=-2x-5$ c) $x-3x+5=x^2-3x+5$	Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal 8. $x-3x+5=-3x^2+5x$ 9. $x-3x+5=-2x-5$ 10. $x-3x+5=x^2-3x+5$
Activity 4: Simplify a) $x-8(x+6)=$ b) $(x-8)x+6=$ c) $(x-3)(x+3)=$ d) $(x-3)-(x+3)=$	Activity 4: Simplify a) $x-8(x+6)=$ b) $(x-8)x+6=$ c) $(x-3)(x+3)=$ d) $(x-3)-(x+3)=$	Activity 4: Simplify a. $x-8(x+6)=$ b. $(x-8)x+6=$ c. $(x-3)(x+3)=$ d. $(x-3)-(x+3)=$
Activity 5 (Post-Test): Simplify a) $2p-(4+p)=$ b) $2p(-4+p)=$ c) $(2+p)+(-4+p)=$	Activity 5 (Post-Test): Simplify a) $2p-(4+p)=$ b) $2p(-4+p)=$ c) $(2+p)+(-4+p)=$	Activity 5 (Post-Test): Simplify a) $2p-(4+p)=$ b) $2p(-4+p)=$ c) $(2+p)+(-4+p)=$

Identifying change moments

- What changes from one lesson to the next?
- How do these changes come about?
- What do we learn about doing Lesson Study?

Reflections on Lesson 1

- Teacher: I stuck to our plan, but then I didn't deal with some of the learner difficulties
- Teacher: I think the lesson overall was too easy, learners "got the idea" and so activity 3 too easy
- The group worked on a different activity for the revised lesson

Thembi: Remember … activity three … they all answered "yes it will make a difference" … already they picked up from activity one and two that … if the bracket is put in a different place it changes the solution. … But for me … that was not enough …

……

Linda: I had another suggestion … what if you … gave them the answer and then said: where must I put the brackets to get this answer?

Thembi: …where you give them x minus three x plus five but then give them the solution and ask them where should we put the brackets? That will be, yes I think that's brilliant. ….

Lesson goal: Learners can simplify expressions with brackets when these are in different positions.

Lesson 2

A small change to the task increased the cognitive demand

This is a more difficult task

Activity 3: insert bracket(s) in the expressions on the left side so that the two sides are equal

1. $x - 3x + 5 = -3x^2 + 5x$
2. $x - 3x + 5 = -2x - 5$
3. $x - 3x + 5 = -x^2 - 3x + 5$

What changes, how, why?

- Changes are a collective accomplishment of both the teachers and the WMCS teacher educators/researchers
  - Reinforcing role of knowledgeable other(s)
- Changes are a function of the MTF framework and so theoretically informed
- Each change is an opportunity for learning mathematics (for) teaching – professional development

In conclusion – our PD

- Grounded in the realities of the schools and classrooms we were working with
- Theoretically informed (a guiding framework for mathematics teaching)
- Impacted teachers' knowledge (and practice) and learner attainment
- A relevant model of lesson study possible, needs time, knowledgeable other, and guiding framework
- Each of these enabled us to improve the project and its models over time

THANK YOU!

KE A LEBOGA!
NGIYABONGA!

DANKIE!
!