Mathematical flexibility:
A promising focus for research and practice

HARVARD GRADUATE SCHOOL OF EDUCATION

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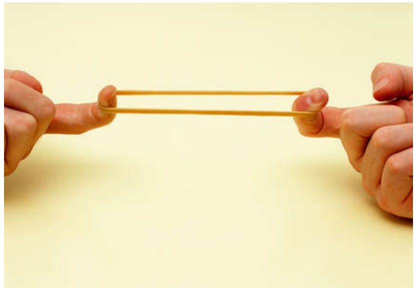
Structure of this talk

1. Definitional considerations
2. Assessing flexibility
3. Recent empirical results on flexibility
4. Promising areas for future flexibility research

1. Definitional considerations

What is flexibility? How is it defined / operationalized?

Flexibility -



- An anatomical attribute - the ability to perform certain kinds of stretches or yoga positions
- A feature of materials – where it is possible to easily bend something without breaking

Flexibility is generally concerned with

- Responsiveness to changing conditions
- Ability to adapt to new circumstances
- Willingness to modify approach when necessary
- Avoidance of rigidity
- Lack of persistence with ineffective or inefficient approaches

Flexibility in *problem solving*

- Willingness to modify problem solving strategies when faced with challenging problems, problem-solving difficulties or failure
- Willingness to change strategies based on the particular problem-solving conditions or goals
- Goals can include efficiency, avoidance of errors, rigor, elegance, ease

Examples

$$3(x+1) = 15$$

Standard algorithm

$$3(x+1) = 15$$
$$3x + 3 = 15$$
$$3x = 12$$
$$x = 4$$

Better strategy?

$$3(x+1) = 15$$
$$x + 1 = 5$$
$$x = 4$$

$$3(x+1) = 14$$

Better strategy?

Standard algorithm

$$3(x+1) = 14$$
$$3x + 3 = 14$$
$$3x = 11$$
$$x = 11/3$$

$$3(x+1) = 14$$
$$x + 1 = 14/3$$
$$x = 11/3$$

Examples

$$4(x + 2) + 3(x + 2) = 21$$

Standard algorithm

$$4(x + 2) + 3(x + 2) = 21$$
$$4x + 8 + 3x + 6 = 21$$
$$7x + 14 = 21$$
$$7x = 7$$
$$x = 1$$

Better strategy?

$$4(x + 2) + 3(x + 2) = 21$$
$$7(x + 2) = 21$$
$$x + 2 = 3$$
$$x = 1$$

$$\frac{5}{3}+\frac{5}{9}+\frac{1}{3}+\frac{4}{9}$$

Examples

Worse than standard algorithm?

Standard algorithm

Method B

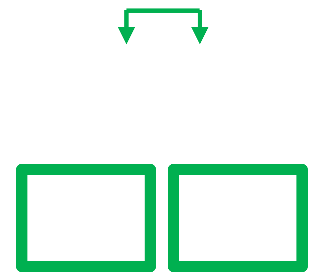
$$\frac{5}{3}+\frac{5}{9}+\frac{1}{3}+\frac{4}{9}$$

$$\frac{15}{9}+\frac{5}{9}+\frac{3}{9}+\frac{4}{9}$$

$$\frac{27}{9}$$

$$3$$

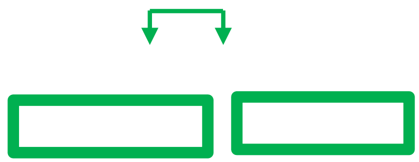
Better strategy?



146 + 12 – 46 + 88

Examples

Better strategy



Standard algorithm

Method B

146 + 12 – 46 + 88

(146 + 12) – 46 + 88

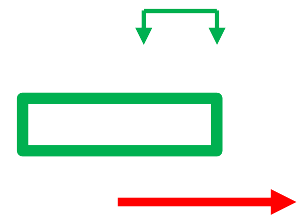
158 – 46 + 88

112 + 88

200

Worse than Better strategy?

Better than Standard algorithm?



A flexible problem-solver:

- While attending to the structural features of a problem, considers which strategies *could* be used and which strategies *should* be used for the problem, taking into account the problem-solving goals (e.g., efficiency)
- Implements the most appropriate strategy for the problem

- Knowledge of multiple strategies

- Ability to select the most appropriate strategy for a given problem

Star, 2005, 2007

Reformulation of an old construct

- Early seminal work (Krutetskii, 1976; Wertheimer, 1959)
- Related to adaptive expertise (Baroody & Dowker, 2003; Hatano & Inagaki, 1986)
- European tradition of studying children's adaptive strategy choices (Blöte, Klein, & Beishuizen, 2000; Torbeyns, Verschaffel, & Ghesquiere, 2006)

Other theoretical considerations

- Influenced by conceptual knowledge and procedural knowledge framework (Hiebert, 1986)

Table 1
Types and Qualities of Procedural and Conceptual Knowledge

Knowledge type	Knowledge quality	
	Superficial	Deep
Procedural	Common usage of *procedural knowledge*	?
Conceptual	?	Common usage of *conceptual knowledge*

(Star, 2005)

Flexibility

- Knowledge of multiple strategies
- The ability to select the most appropriate strategy for a given problem

Other theoretical considerations

- Influenced by conceptual knowledge and procedural knowledge framework (Hiebert, 1986)
- Relates to psychological distinction between **competence** and **performance** (Flavell & Wohlwill, 1969; Le Corre et al., 2006)
  - Students have knowledge of strategies but do not consistently utilize this knowledge during problem solving performances
  - Competence tends to precede performance

2. Assessment of flexibility

How can mathematical flexibility be assessed?

Knowledge of multiple strategies and the ability to select the most appropriate strategy for a given problem / problem-solving circumstance

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Ask learners to solve problems, often in multiple ways

- Generate their own strategies to directly indicate procedural flexibility

Examples *Successes* *Challenges*

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Kim solved the following problem:

$$\frac{1}{3}(x+5)=4$$

Kim's first step was:

$$(3)\frac{1}{3}(x+5)=4(3)$$
$$x+5=12$$

What step was used to get from the first line to the second line?

a) Combine like terms
b) Distribute across parentheses
c) Add or subtract the same quantity to both sides
d) Multiple or divide the same quantity to both sides

Knowledge of multiple strategies

Do you think that this is a good way to start this problem?

a) Very good way
b) OK, but not a very good way
c) Not OK

Ability to select the most appropriate strategy

Examples

Rittle-Johnson & Star, 2007

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

28) On a timed test, which would be the best way to **start** this problem?
(Choose the letter for the best way to start.)

$$3(x+2)=14$$

a. Anna's "distribute first" way:	b. Ben's "divide by 3 on both sides first" way:	c. Chris's "multiply by 3 on both sides first" way:	d. Drew's "subtract 14 from both sides first" way:
$3(x+2)=14$ $3x+6=14$	$\frac{3(x+2)}{3}=\frac{14}{3}$ $x+2=\frac{14}{3}$	$(3)3(x+2)=14(3)$ $9(x+2)=42$	$3(x+2)=14$ $-14\ -14$ $3(x+2)-14=0$

Ability to select the most appropriate strategy

Examples

Rittle-Johnson & Star, 2007

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Select the best solution:

4、 $2(X+3)+3(X+3)=20$

Solution①:	Solution②:	Solution③:
$2X+2\times3+3X+3\times3=20$	$5（X+3）=20$	$5（X+3）=20$
$5X+15=20$	$5X+5\times3=20$	$X+3=\frac{20}{5}$
$X=1$	$5X=20-15$	$X=1$
	$X=1$	

Ability to select the most appropriate strategy

Examples

Xu, Liu, Star, Wang, Liu, & Zhen, 2017

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Method A	Method B	Method C
$3(x + 1) = 15$	$3(x + 1) = 15$	$3(x + 1) = 15$
$3x + 3 = 15$	$x + 1 = 5$	$3x + 3 = 15$
$3x = 12$	$x = 4$	$3x + 3 - 15 = 0$
$x = 4$		$3x - 12 = 0$
		$3x = 12$
		$x = 4$

(iv) How **GOOD** is **Method A** for solving this problem?

Not Very Good — Indifferent — Very Good

(v) How **GOOD** is **Method B** for solving this problem?

Not Very Good — Indifferent — Very Good

(vi) How **GOOD** is **Method C** for solving this problem?

Not Very Good — Indifferent — Very Good

Ability to select the most appropriate strategy

Examples

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Successes

- Process of writing and iteratively improving forced choice response questions has been very instrumental in sharpening our thinking around procedural flexibility and how to measure it.

Challenges

- These types of questions require learners to make sense of another's strategy
- Are these items measuring procedural flexibility and/or learners' ability to make sense of another's strategies?

Ask learners to answer forced choice response questions

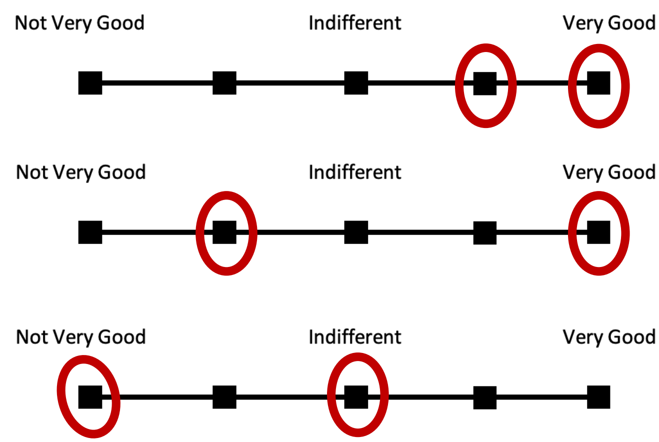
- React to or analyze others' strategies to indirectly indicate procedural flexibility

Method A	**Method B**	**Method C**
3(x + 1) = 15 3x + 3 = 15 3x = 12 x = 4	3(x + 1) = 15 x + 1 = 5 x = 4	3(x + 1) = 15 3x + 3 = 15 3x + 3 – 15 = 0 3x – 12 = 0 3x = 12 x = 4

(iv) How **GOOD** is **Method A** for solving this problem?

(v) How **GOOD** is **Method B** for solving this problem?

(vi) How **GOOD** is **Method C** for solving this problem?



Examples

Ask learners to solve problems, often in multiple ways

- Generate their own strategies to directly indicate procedural flexibility

- Solve a problem, and later re-solve the same problem but using a different way e.g., Star & Seifert, 2006; Rittle-Johnson & Star, 2007

Knowledge of multiple strategies

- Solve a problem, and later re-solve the problem but using as many different ways as you can think of

Knowledge of multiple strategies

e.g., Xu, Liu, Star, Wang, Liu, & Zhen, 2017

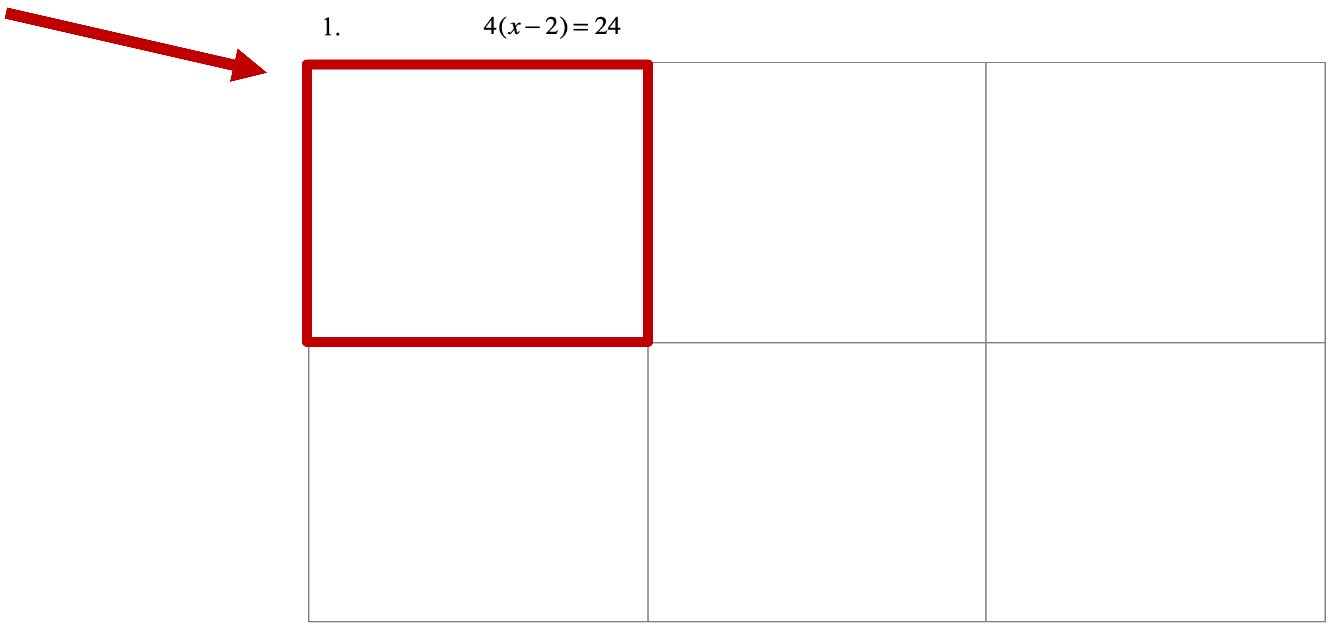
Examples

Ask learners to solve problems, often in multiple ways

- Generate their own strategies to directly indicate procedural flexibility

- When learners are asked to solve a problem in several different ways, the strategy that is used first is an implicit indicator of which strategy the learner views as the best

1. $4(x-2)=24$



Ability to select the most appropriate strategy

Examples

Ask learners to solve problems, often in multiple ways

- Generate their own strategies to directly indicate procedural flexibility

Successes

- Asking learners to use a different strategy appears to provide a good measure of their knowledge of multiple strategies
- May also push students to become more flexible?

Challenges

- To what extent do students' beliefs about what it means for a strategy to be DIFFERENT or BEST affect their strategy generation?

Knowledge of multiple strategies and the ability to select the most appropriate strategy for a given problem / problem-solving circumstance

Ask learners to answer forced choice response questions

- React to or analyze others' strategies to indirectly indicate procedural flexibility

Ask learners to solve problems, often in multiple ways

- Generate their own strategies to directly indicate procedural flexibility

Integrate learner-generated strategies with forced-choicer response questions

- # Tri-phase flexibility assessment

(Xu et al., 2017; Liu et al., 2018)

- Students are given 12 problems to solve:

  Phase One: Solve each problem quickly and accurately.

  Phase Two: Solve each problem again, in as many different ways as possible.

  Phase Three: Select the one strategy that you felt was best for each problem and circle it.

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frontiers in Psychology

Measures of Potential Flexibility and Practical Flexibility in Equation Solving

ORIGINAL RESEARCH
published: 10 August 2017
doi: 10.3389/fpsyg.2017.01368

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Operationalization of flexibility

Phase 1: Solve each problem quickly and accurately.

Phase 2: Solve each problem again, in as many different ways as possible.

Phase 3: Select the one strategy that you felt was best for each problem and circle it.

On a given problem, a student is flexible if he/she:

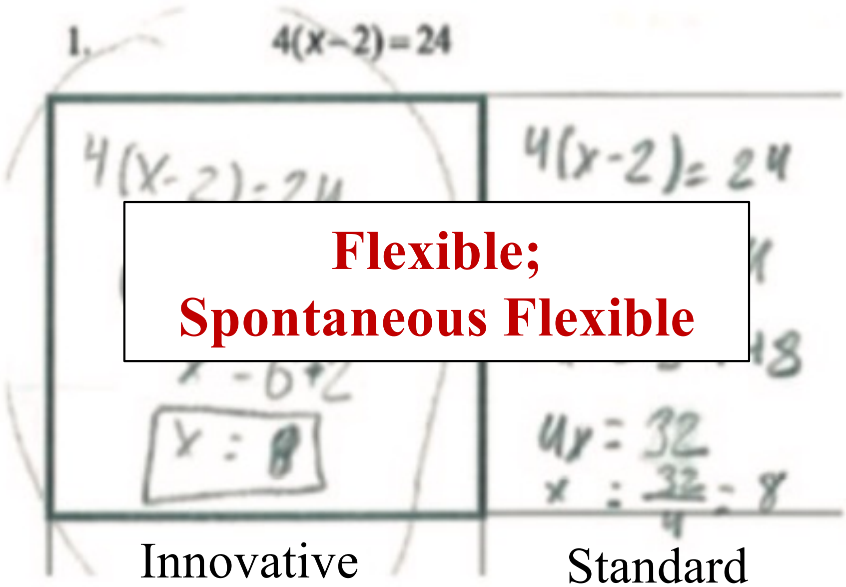
A. Uses a standard strategy
B. Uses an innovative strategy } Phases 1 or 2
C. Identifies (circles) the more innovative strategy → Phase 3

- Potential flexibility
  - Two of these three criteria (AB, BC, or AC)
- Spontaneous flexibility
  - The innovative strategy is used on the first attempt → Phase 1

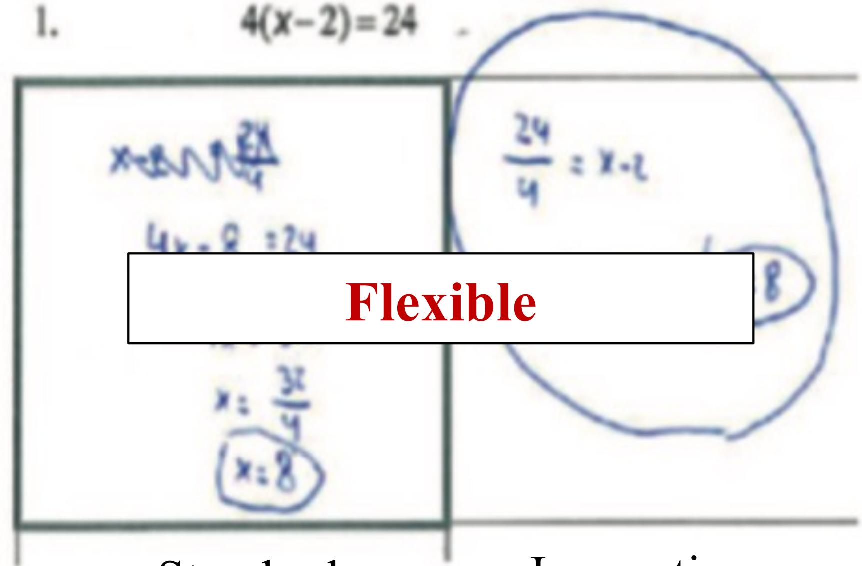
$$4(x - 2) = 24$$

$4(x - 2) = 24$ $4x - 8 = 24$ $4x = 32$ $x = 8$	$4(x - 2) = 24$ $x - 2 = 6$ $x = 8$

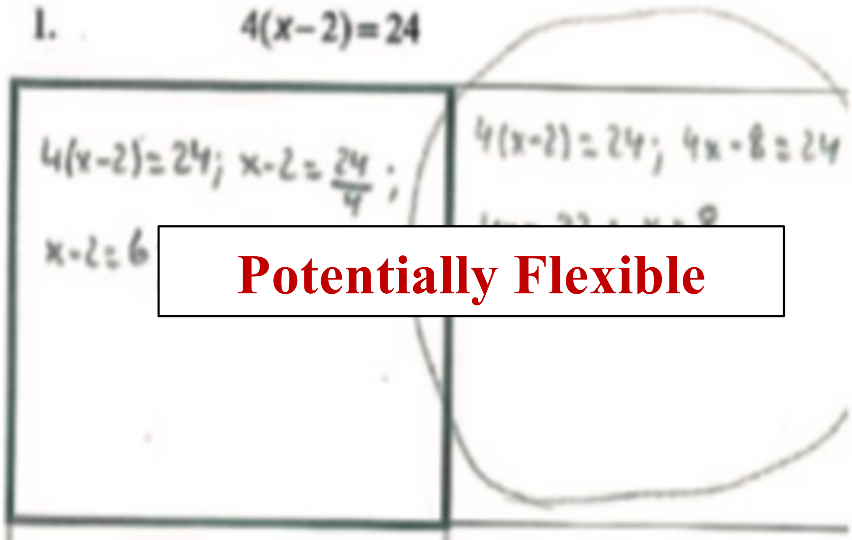
For example (from Joglar, Abánades, & Star, 2018):



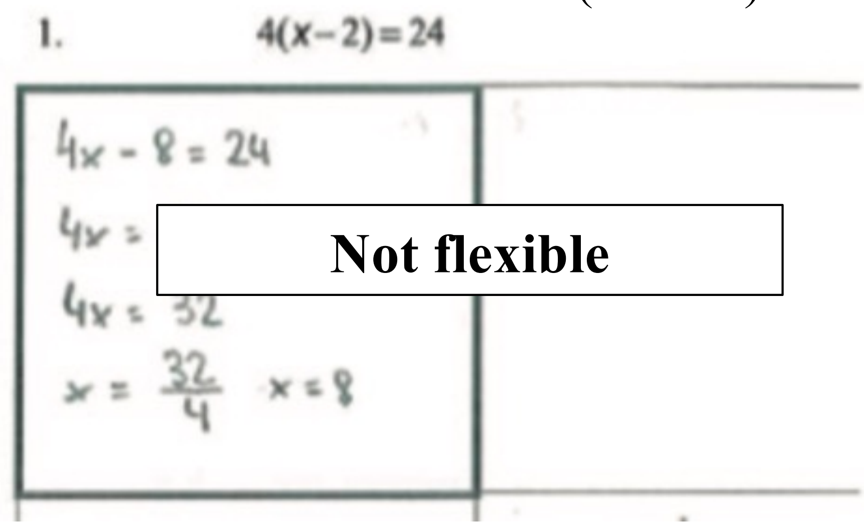
Innovative (Circled) | Standard



Standard | Innovative (Circled)

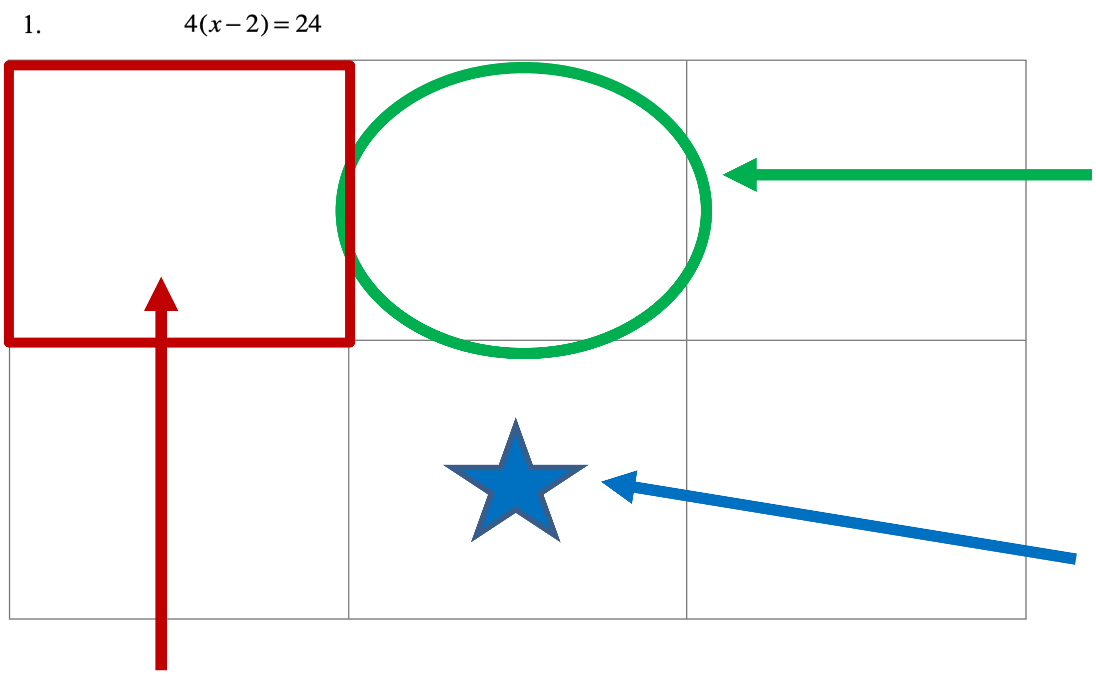


Innovative | Standard (Circled)



Standard | Blank

More recently….



Circled strategy *(Forced-choice)*

- In a subsequent phase of the assessment (and after generating multiple strategies), students explicitly circled the strategy that they believed was the best

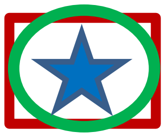
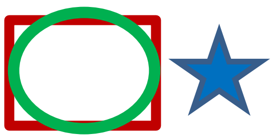
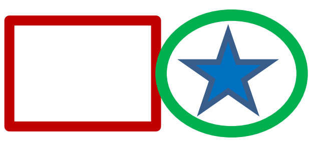
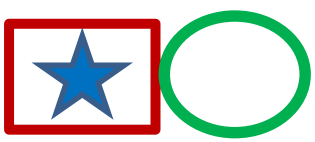
Expert judgement

- Experts' decision as to which strategy is the best, *of the strategies that the student provided*

First box strategy *(Generation)*

- Assumption: What a student writes in the first box is an implicit indication of which strategy they believe is the best for this problem

Star et al., in press; Jiang et al., in press

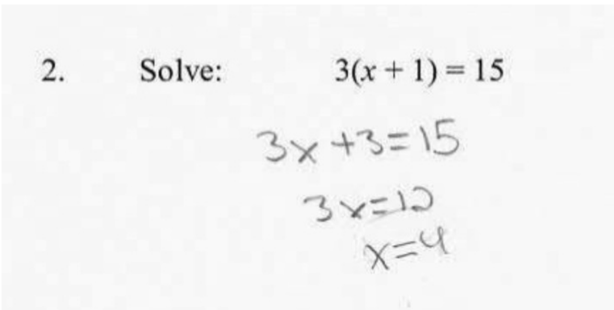
			Percent of problems where students provided multiple strategies
	First box strategy = circled strategy = expert best strategy	Internally consistent Expert aligned	45
	First box strategy = circled strategy, but **not** best by experts	Internally consistent NOT expert aligned	11
	Expert strategy is circled as best, but **not** put in first box	NOT internally consistent Explicitly expert aligned	38
	Expert strategy is put in first box, but **not** circled	NOT internally consistent Implicitly expert aligned	6

Sample: 792 middle school and high school students in Finland, Spain, and Sweden

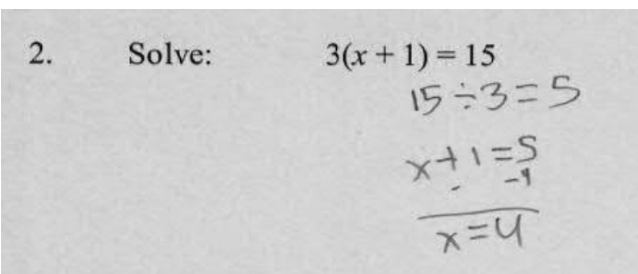
Jiang et al., in press

A final assessment example…

Part I: “Solve …”



Part II: ”Solve using a DIFFERENT method than the one you used before.”

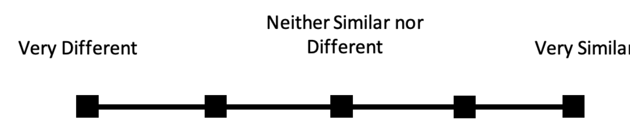


(Generation)

Method A	**Method B**	**Method C**
$3(x+1) = 15$	$3(x+1) = 15$	$3(x+1) = 15$
$3x + 3 = 15$	$x + 1 = 5$	$3x + 3 = 15$
$3x = 12$	$x = 4$	$3x + 3 - 15 = 0$
$x = 4$		$3x - 12 = 0$
		$3x = 12$
		$x = 4$

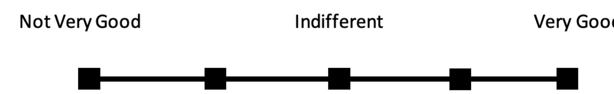
Part III:

(ii) How **SIMILAR** is **Method B** to the method that you **FIRST** used?



…

(v) How **GOOD** is **Method B** for solving this problem?



(Forced-choice)

3. Empirical results

What have we learned recently about flexibility?

Recent empirical results

1. Can flexibility be influenced by curriculum and instruction?
2. What is the relationship between flexibility and accuracy?
3. Are there cross-cultural differences in flexibility and its development?

1. Influenced by curriculum / instruction?

- A curricular and/or instructional focus on flexibility can lead to improvements (Star, Pollack et al., 2015; Star, Newton et al., 2015)
- We have designed a supplemental algebra curriculum that can be used to promote the development of flexibility, as well as conceptual and procedural knowledge.
- Here are some examples from the curriculum:
  - See www.compareanddiscuss.com for more examples

Emma and Layla were asked to provide a second equation so that the following system of equations has no solution.

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

Emma's "graphing" way

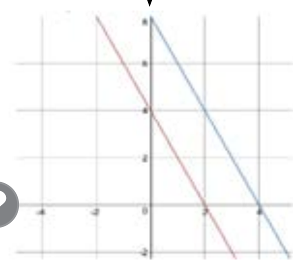
I found the x- and y-intercepts of the first equation and graphed the line.

Then, I graphed a line parallel to it.

I wrote the equation of this new line.

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

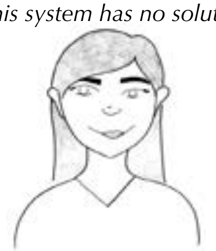
4x + 2y = 8
x-intercept = 2
y-intercept = 4



y = -2x + 8

$$\begin{cases} 4x + 2y = 8 \\ y = -2x + 8 \end{cases}$$

This system has no solution



Layla's "using algebra" way

I changed the first equation to slope-intercept form.

I then found the slope and y-intercept.

I wrote a new equation with a different y-intercept.

$$\begin{cases} 4x + 2y = 8 \\ \end{cases}$$

4x + 2y = 8
2y = -4x + 8
y = -2x + 4

m = -2
b = 4

y = -2x + 8

$$\begin{cases} 4x + 2y = 8 \\ y = -2x + 8 \end{cases}$$

This system has no solution

Why did Emma graph a parallel line? How did Layla come up with her new equation?

How did Emma and Layla come up with the same second equation? Are there other second equations that would have worked?

Gloria and Tim were solving the problem
f(x) = 4x + 1
to find f(2).

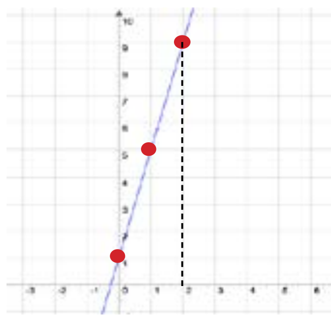
Gloria's "graphing" way

The equation is in slope-intercept form, so I know that 4 is the slope and 1 is the y-intercept.

I plotted the y-intercept then continued to plot points using the slope.

I got (2,9) as my answer.

f(x) = 4x + 1



(2,9)

Tim's "function notation" way

f(x) = 4x + 1
f(2) = 4(2) + 1
f(2) = 8 + 1

f(2) = 9

I am solving for the ouput, and I know 2 is the input.

I got f(2)= 9 as my answer.

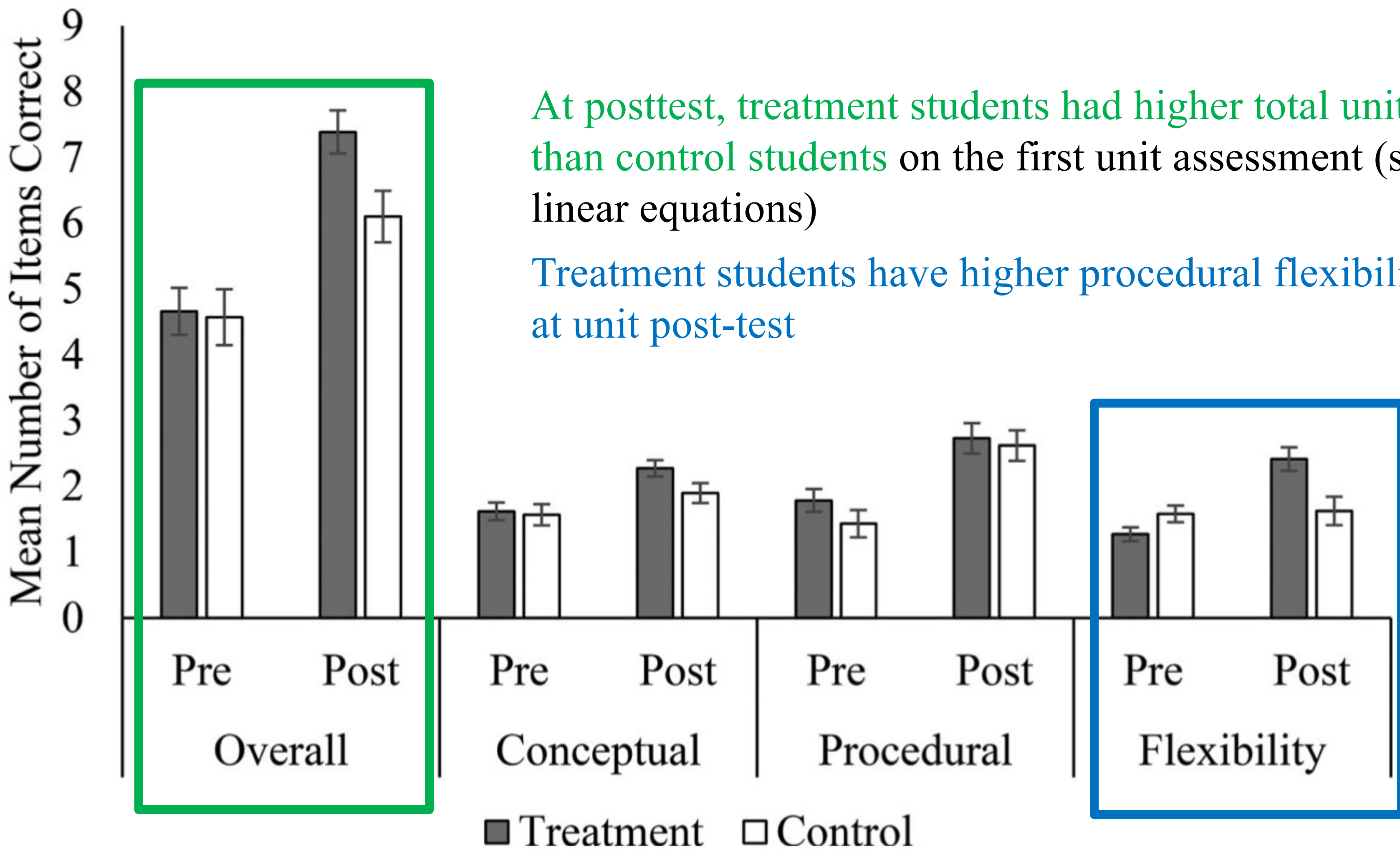
How did Gloria know to find 2 on the x-axis instead of the y-axis?

Did Gloria and Tim get the same answer? How do you know?

2021 Study

- 15 Algebra I teachers and their 431 students
  - Received summer and school-year professional development
  - Taught with the supplemental curriculum
- 12 Algebra teachers and their 289 students were 'business-as-usual' controls
- Unit pre- and post-tests, focusing on conceptual knowledge, procedural knowledge, and flexibility
- Teachers were videotaped to assess fidelity of implementation

Promising evidence of success



At posttest, treatment students had higher total unit scores than control students on the first unit assessment (solving linear equations)

Treatment students have higher procedural flexibility at unit post-test

Durkin, Rittle-Johnson, Star, & Loehr (2021)

2. Flexibility and accuracy?

- Are more flexible problem solvers also more accurate problem solvers?

Standard algorithm generally more accurate? → Yes: automatic, routine execution leads to fewer errors?

'Better' strategies generally more accurate? → Yes: fewer steps; require more deliberate and conscious attention

Perhaps it depends, on … → Ease of 'seeing' the better strategy?
How 'helpful' the better strategy is?
The mathematical domain?

- 450 high school students in the US
- Solve these 5 "flexibility-eligible" problems
- Then re-solve, using a different way

#	Problem	Standard algorithm	Better strategy?
1	Simplify: $\frac{5}{3}+\frac{5}{9}+\frac{1}{3}+\frac{4}{9}$	Use common denominator of 9	Add 3rds and 9ths
2	Solve: $3(x+1)=15$	Distribute first	Divide by 3 first
3	Solve: $4(x+2)+3(x+2)=21$	Distribute first	Combine $(x+2)$ terms
4	Simplify: $\frac{1}{5}\times\frac{13}{10}+\frac{13}{10}\times\frac{4}{5}$	Compute left to right	Factor our 13/10 first
5	Simplify: $146+12-46+88$	Compute left to right	Commute for easier computation

	All (N=2,811)		Solve first time (n=1,669)		Solve second time (n=1,142)	
	Stnd	Better	Stnd	Better	Stnd	Better
Incorrect	382 (21.8%)	313 (29.7%)	243 (19.0%)	139 (35.6%)	139 (29.1%)	174 (26.2%)
Correct	1,374 (78.3%)	742 (70.3%)	1,035 (81.0%)	252 (64.5%)	339 (70.9%)	490 (73.8%)
Total	1,756	1,055	1,278	391	478	664

Standard more accurate than Better when solving the first time, but not for solving the second time

Standard more accurate when solving first time than for second time

Better more accurate when solving second time than for first time

Nuanced conclusions

- We cannot conclude that the standard algorithm is generally more accurate, nor can we conclude that better strategies are more accurate.
- The relationship between flexibility (choice of strategy, standard or better) and accuracy is nuanced and depends a lot on the problem, the problem domain, and the assessment task (solving for the first time or the second time)
- More work is needed to understand this relationship

3. International study on flexibility

- Over 800 students from Finland, Sweden, and Spain completed the Tri-phase flexibility assessment
- Middle school students = 8^{th} grade (~ age 14)
    - Advanced and regular
- High school students = 11^{th} grade (~ age 17)
    - Advanced and regular
- Cross-sectional design

#	Problem	Initial steps of the standard algorithm		Initial steps of a situationally appropriate strategy	
1	$4(x-2)=24$	Begin by distributing the parentheses	$4x-4\cdot 2=24$	Divide a constant to both sides before distributing	$x-2=\frac{24}{4}$
2	$3(x+0.69)=15$		$3x+3\cdot 0.69=15$		$x+0.69=\frac{15}{3}$
3	$4\left(x+\frac{3}{5}\right)=12$		$4x+4\cdot\frac{3}{5}=12$		$x+\frac{3}{5}=\frac{12}{4}$
4	$4(x+6)+3(x+6)=21$	Begin by distributing the parentheses	$4x+4\cdot 6+3x+3\cdot 6=21$	Change in variable – combine	$7(x+6)=21$
5	$5\left(x+\frac{3}{7}\right)+3\left(x+\frac{3}{7}\right)=16$		$5x+5\cdot\frac{3}{7}+3x+3\cdot\frac{3}{7}=16$		$8\left(x+\frac{3}{7}\right)=16$
6	$2(x-0.31)+3(x-0.31)=15$		$2x-2\cdot 0.31+3x-3\cdot 0.31=15$		$5(x-0.31)=15$
7	$8(x-5)=3(x-5)+20$	Begin by distributing the parentheses	$8x-8\cdot 5=3x-3\cdot 5+20$	Change in variable – subtract from both	$5(x-5)=20$
8	$8\left(x-\frac{2}{5}\right)-11=6\left(x-\frac{2}{5}\right)$		$8x-\frac{16}{5}-11=6x-\frac{12}{5}$		$2\left(x-\frac{2}{5}\right)=11$
9	$5(x+0.6)+3x=5(x+0.6)+7$		$5x+3+3x=5x+3+7$		$3x=7$
10	$\frac{2x-6}{2}+\frac{6x-18}{3}=5$	Begin by obtaining a common denominator for the two expressions	$\frac{3(2x-6)}{2\cdot 3}+\frac{2(6x-18)}{2\cdot 3}=5$	Reducing each fraction before combining	$(x-3)+(2x-6)=5$
11	$\frac{x+3}{3}+\frac{3x+9}{9}=1$		$\frac{3\cdot(x+3)}{3\cdot 3}+\frac{3x+9}{9}=1$		$\frac{x+3}{3}+\frac{x+3}{3}=1$
12	$\frac{5x+5}{5}+\frac{6x+6}{6}=6$		$\frac{6(5x+5)}{6\cdot 5}+\frac{5(6x+6)}{5\cdot 6}=6$		$(x+1)+(x+1)=6$

	Used Standard Algorithm on at least one problem			Used Better strategy on at least one problem		
	Middle School (all)	High School (reg)	High School (adv)	Middle School (all)	High School (reg)	High School (adv)
Finland	61.3	95.1	100.0	40.9	42.6	93.2
Sweden	96.6	-	98.9	50.6	-	93.9
Spain	98.2	100.0	100.0	22.6	60.0	68.1

Standard algorithms widely used across all countries and ages.

Better strategies widely used among high school students in Finland and Sweden.

Less common for Finnish and Swedish middle school students to use Better strategies.

Spanish students' use of Better strategies is substantially lower than other countries.

Moving forward

4. Promising areas for future flexibility research

Future research on flexibility

1. Expansion of research into other mathematical domains
2. Possibility of transfer of flexibility between domains
3. Clarify relationship between fluency and flexibility

1. Expansion into new domains

- Most of my flexibility work is in algebra (linear equation solving)
- Some work in Calculus (Maciejewski & Star, 2016, 2019) and pending work in geometry
- Other mathematical topics unexplored
- Relatively easy to adapt tri-phase flexibility assessment and other flexibility assessments discussed today to other domains

2. Transfer of flexibility

- If a student is flexible in linear equation solving, is he/she also flexible in other mathematical domains? In non-mathematical domains?
- Other than psychological studies showing that transfer is very difficult to achieve, the possibility of transfer of flexibility has not been explored
- Why might flexibility transfer? Is there a metacognitive component to flexibility? Is flexibility a ‘mindset’?

3. Flexibility and fluency

- The developmental relationship between flexibility and fluency needs clarification
- If we want students to become flexible, to what extent is fluency a prerequisite?
- Or does fluency impede the development of flexibility?
- Some prior work has touched on this issue (e.g., Rittle-Johnson, Star, & Durkin, 2012), but this relationship is under explored

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Thank you!

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