ASIAN CENTRE FOR MATHEMATICS EDUCATION
华东师范大学·亚洲数学教育中心

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From studying students' learning to studying teaching: Some experiences from local and international projects
从研究学生的学习到研究教学：国内外项目的一些经验

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Abstract 摘要:

The talk consists of a synthesis of some work from three research projects.

1. A study of students' understanding of the distributive property (My PhD study)
2. The Learner's Perspective Study (An international collaboration)
3. Integrating Research into Practice: The Growth of Collective Pedagogical Content Knowledge (A collaboration with my doctoral student)

- Via these projects, I would like to share my experiences and interests for research in students' conceptions, classroom teaching and collaboration with teachers.

本次演讲综合了三个研究项目的一些工作。

1. 学生对乘法分配律理解的研究（我的博士论文研究）
2. 学习者的视角研究（国际合作）
3. “将研究融入实践：集体教学内容知识(Collective PCK)的进程（与我的博士生合作）

- 通过这些项目，我想分享我在学生概念研究、课堂教学研究以及与教师合作研究方面的经验和兴趣。

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A study of students' understanding of the distributive property

(My PhD study)

80's and 90's

学生对乘法分配律理解的研究
(我的博士论文研究)
80年代和90年代

Research Focus: How do the students understand the distributive law?

Common students' errors that shared the distributive pattern

- The interest of this research originated from the observation of a set of common students' errors that shared the distributive pattern. For example,
- "$(t+1)^2 = (t^2+1)$" $[= t^2+1^2]$
- "$5(x^2)(2xy) = (5x^2)(10xy)$" $[=(5\times x^2)(5\times 2xy)]$
- "$\cos 90^o - \cos 30^o = \cos 60^o$" $[= \cos(90^o - 30^o)]$

Theoretical Basis

- Constructivism: Students' errors or misconceptions provide a window for interpreting their understanding in the learning process.
- SOLO taxonomy: A classification for Students' Observed Learning Outcomes
- Grounded Theory Approach

Baroody Arthur J., Ginsburg Hiebert P., 1990, "Children's learning: A Cognitive View", in Davies Robert B., Maher Carolyn A., and Noddings Nel (eds.) Constructivist Views on the Teaching and Learning of Mathematics, Journal for Research in Mathematics Education, monograph number 4, NCTM.

Biggs, J. B., & Collis, K. F. (1982). Evaluating the quality of learning: The SOLO taxonomy (Structure of the observed learning outcome). New York, London: Academic Press.

Strauss, A., & Corbin, J. M. (1997). Grounded theory in practice. Sage.

Design of the study 研究设计

- The study was an investigation of the students' understanding of the distributive property of a group of students aged 12-18.
- A secondary school of average standard in Hong Kong
- Secondary 1(6) , 2(6), 3(6), 4(6) & 4 Sc(3); secondary 6(6)
- 51 task-based clinical interviews for 33 students.

- 本研究调查了12-18岁学生对分配律的理解。(1994-1996)
- 香港中学(一般水平）。
- 中1(6) ,中2(6),中3(6),中4(6) &中4理(3); 中6(6)
- 对33个学生51次以任务为基础的诊断访谈。

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Significance of the Research
研究重点

- Focus on students' understanding of one algebraic properties
- Alignment with Hong Kong curriculum
- Development of a diagnostic instrument for task-based interview
- Theoretical Perspectives: Constructivism, Application of SOLO taxonomy
- A cross-sectional study of secondary 1, 2, 3, 4 and 6

- 注重学生对一个代数性质的理解
- 与香港课程保持一致
- 任务型访谈诊断工具的开发
- 理论观点：建构主义, SOLO分类法的应用
- 中一、二、三、四和六年级的横断面研究

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Research Instruments. 研究工具：

- A written test designed by the researcher.
- The Chelsea Diagnostic Algebra Test
- Six interview tasks designed by the researcher

- 由研究人员设计的笔试。
- 切尔西诊断代数测试
- 研究者设计的六项访谈任务

Hart, K. (Ed.) (1981). Children's Understanding of Mathematics: 11-16. 102-(19). London: Murray.

What questions will you raise from these percentages?

从以下这些百分比来看，大家会提出什么问题？

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Facility of selected items in the written test 笔试中某些项目的正确率 (true/false)

	S. 1 (40)	S. 2 (39)	S. 3 (36)	S. 4A (38)	S. 4SC (40)	S.6 (10)
3×(4+5)= 3×4+3×5	82.5% (33)	92.3% (36)	72.2% (26)	89.5% (34)	100%	100%
47×(69–23)= 47×69–47×23	62.5% (25)	74.4% (29)	72.2% (26)	73.7% (28)	100%	100%
a(b+c)=ab+ac	75.0% (30)	100% (39)	94.4% (34)	100%	100%	100%
a(b–c)=ab–ac	75.0% (30)	97.4% (38)	94.4% (34)	92.1% (35)	100%	100%
a÷(b+c)= a÷b+a÷c	57.5% (23)	35.9% (14)	41.7% (15)	55.3% (21)	80.0% (32)	90% (9)
a÷(b–c)= a÷b–a÷c	57.5% (23)	41% (16)	36.1% (13)	63.2% (24)	85.0% (34)	90% (9)
$(a-b)^2=a^2-b^2$	52.5% (21)	30.8% (12)	30.6% (11)	94.7% (36)	100%	100%
$5(a^2)(2b) = (5a^2)(10b)$	55.5% (22)	43.6% (17)	44.4% (16)	81.6% (31)	97.5% (39)	100%
$(ab)^2=a^2b^2$	72.5% (29)	79.5% (31)	100%	97.4% (37)	100%	100%
$(ab)^n=a^nb^n$	75.0% (30)	82.1% (32)	100%	100%	100%	100%

SOLO taxonomy (Biggs & Collis, 1982) Students' Observed Learning Outcomes

- Prestructural: not engaged in the task, ignore, do something else
- Unistructural: focus on one single aspect, end the answer very quickly
- Multistructural: multiple aspects but not necessarily coherent
- Relational: coherent, consistent
- Extended Abstract: hypothetical, verification, generalization

Biggs, J. B., & Collis, K. F. (1982). Evaluating the quality of learning: The SOLO taxonomy (Structure of the observed learning outcome). New York, London: Academic Press.

The interview items (4 items applying SOLO taxonomy, 2 items on common students' mistakes)

Card 1

Is 62× (23+49) = 62×23+62×49 correct? Why?

Card 2

When will (a+b) ÷ c=a÷c+b÷c be true? Why?

always ___ never ___

sometimes when ______

Reasons: :

Card 3

When will a÷ (b+c)=a÷b+a÷c be true? Why?

always ___ never ___

sometimes when ______

Reasons :

Card 6

If a, b, c stand for any numbers, □ and ■ stand for any of the operations +,–, × and ÷, when will a□(b■c)=a□b■a□c be true?

always ___ never ___

sometimes when ___________________

Reasons:

Card 4

Assume that you are a mathematics teacher, comment on the following.

$5(x^2)(2xy) = (5x^2)(10xy)$

$(t+1)^2 = t^2+1$

$\cos 90^o - \cos 30^o = \cos(90^o-30^o)$

$\cos 90^o - \cos 30^o = 30(\cos 3^o-\cos 1^o)$

$(ab)^n=a^n b^n$

Card 5

Are the following correct? Tick or cross.

a × b ÷ a × c = (a × b) ÷ (a × c)

a ÷ b × a ÷ c = (a ÷ b) × (a ÷ c)

a ÷ b ÷ a ÷ c = (a ÷ b) ÷ (a ÷ c)

a+b × a+c = (a+b) × (a+c)

a–b × a–c = (a–b) × (a–c)

In addition to applying SOLO taxonomy, the analysis also applied:

- Other research findings in students' understanding of algebra, based on the literature review.
- Grounded theory approach.

Ideas of 'algebra' performing syntax
transformation
replacing unknowns by numbers

Interpretation of letters and symbols
letters represented numbers
different symbols represented
different entities
different squares represented different operations
different letters represented different numbers
misconception

Structure of numbers, symbols and operations
meaning of power
misconception division was commutative

Syntactic actions based on syntax and structure
Decision-making rules of priority of arithmetic operations
"equal" meant identical syntax
misconception such as different procedures meaning unequal answers

Booth Lesley R., 1984, Algebra: Children's Strategies and Errors. A Report of the Strategies and Errors in Secondary Mathematics Project, Windsor, Berkshire: NFER-NELSON.

Watson, A. (2009). Algebraic reasoning. In A review commissioned by the Nuffield Foundation, Key understandings in mathematics learning. Nuffield Foundation. http://www.nuffieldfoundation.org/sites/default/files/P6.pdf

An Example of coding the transcript:
Is 62∗(23+49) = 62∗23+62∗49 correct? Why?

I: How can you know that they are equal?

John: Because, for algebra, sixty-two, er, remove bracket, becomes two "sixty-two" then sixty-two times twenty-three, then plus sixty-two times forty-nine. Correct.

concept/ category: algebra domain, remove bracket, numbers as objects, procedural, syntax transformation, unistructural

coding notes

emphasize "remove bracket", this was a procedure description of syntax transformation.

referred to working in algebra domain but was not clear.

"Two 62's" were mentioned, thus the occurrence of "62" was counted.

comments

What is John's idea of algebra?

Will procedure description be a dominating feature?

Example 1 (unistructural, removing bracket)

Is $62\times(23+49) = 62\times23+62\times49$ correct? Why?

- "I think that [$62 \times(23 + 49) = 62 \times 23 + 62 \times 49$] it is correct because the bracket can be removed. I can remove to 62 times 23 plus 62 times 49"
- a simple recall of a syntactic action of removing brackets.

Example 2 (relational, continued from Example 1, comparing unevaluated multiples)

Is $62\times (23+49) = 62\times23+62\times49$ correct? Why?

- [$62 \times(23 + 49) = 62 \times 23 + 62 \times 49$] removing bracket.

I: Why can the bracket be removed?

S: Because, let me think, because bracket inside, is a group of numbers and they multiply to bigger. 62, taken out, means to divide one group of numbers into two groups. Then times each. The total is 62 times the group of numbers.

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Example 2 (multistructural, removing bracket, fraction representation, formula)

- A student described the given statement *as a result* of ***removing the brackets***.
- She wrote the **fraction representation** and she "**a correct formula**" and she knew that the calculated value for both sides would eventually be the same.

Card 2
When will (a+b) ÷ c=a÷c+b÷c be true?
Why?
always ___ never ___
sometimes when _____
Reasons: :

$$(a+b) \div c = a \div c + b \div c$$

$$\frac{(a+b)}{c} = \frac{a}{c} + \frac{b}{c}$$

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Extended Abstract

1. In EA responses, the student can handle hints, relevant data and hypotheses.
2. The student may demonstrate both inductive and deductive argument in his reasoning.

Card 3
When will a÷ (b+c)=a÷b+a÷c be true? Why?
always ___ never ___
sometimes when ______
Reasons：

Consider cases, e.g., a=0; b=c

Making hypothetical situation, verify with correct mathematics, coherent argument.

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Some examples of analysis of the students' answers in the interviews

分析学生在访谈中的回答的一些例子

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Who will say that this is wrong?

$62 \times (23+49) = 62 \times 23 + 62 \times 49$

- Pui in secondary-one, Ahkit in secondary-two, and Shumang, Heihei and Hoihoi in secondary-three.
- Ahhang (secondary-2) did not know the answer.

1. The two different calculation procedures would give different answers
2. They could not give any reasons.

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Ahhang knew: “Letters represent unknown numbers.”

Ahhang [secondary-2, card 1, 62 × (23+49) = 62 × 23+62 × 49]

I: You said that you did not know whether it was correct. Do you have any methods to find out?

Ahhang: Calculate.

I: Need to “calculate”?

Ahhang: Yeah.

I: If [you] do not calculate, are there any methods? [Probing, to see whether the student could relate the statement to algebra.]

Ahhang: Not calculated, not calculated... [There] may be some methods.

I: Do you know any?

Ahhang: Er.... No... Earlier... I thought of algebra, but [I] don’t know whether these are the same.

I: Tell me. What algebra did you think of?

Ahhang: Those in primary mathematics textbooks. Like this, but replace... letters. That is, this times this, then times this. But [I] don’t know whether it represents this.

I: Yeah. What do the letters in the books represent?

Ahhang: [They] represent, represent unknown numbers

Does he know any algebra?

Procedural descriptions:
Telling how to do it correctly

Ahming [secondary-1, 1st interview, card one]

Ahming: Because, for algebra, sixty-two, er, remove bracket, becomes two "sixty-two" then sixty-two times twenty-three, then plus sixty-two times forty-nine. [It is] correct.

Do they know any more? Will there be other explanations?

Hokhok [secondary-2, card one]

Hokhok: Because, 62 equals, er... 62 times 23 plus... Let me think. How to explain? Er... Because the numbers inside the bracket, there are two. The number outside multiplies the two numbers inside.

Factorization: Still a procedural recall!?

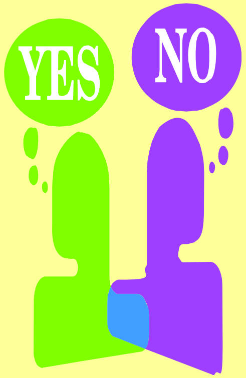
Ahmui [secondary-4A, card 1]

Ahmui: Er. 62 times 23 plus 62 times 49. Er. They both multiply the same number. So take out [factorize] the "62".

Factorization was deemed to be at a higher level as it was taught in later school years.

However, like the remove-bracket responses, explanations in terms of factorization suggested a recall of an application of an algorithm.

That is, recognizing the distributive law at an instrumental level and it gave no further information about the students' relational understanding.



Generalization to Division: Right/wrong ? Why? How to justify? 归纳:对/错？ 为什么？ 如何辩解？

Card 2

When will (a+b) ÷ c=a÷c+b÷c be true? Why?

always ___   never ___

sometimes when _________________

Reasons:

Card 3

When will a÷ (b+c)=a÷b+a÷c be true? Why?

always ___   never ___

sometimes when _________________

Reasons:

How will your students answer these cards?

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Both Cards 2 and 3 were Always Correct.

Ahyi [secondary-2, $a \div (b+c) = a \div b + a \div c$]

Ahyi: Er. Put this, this... this, add, subtract numbers. Em. This a divided by b plus a divided by c, so [it's] two. Em, this is also division, this is also division. Em. Both are a divided by some numbers. The two [are] the same, [I] can take out the common factor, that is, that is b plus c equal. No. a divided by b plus c.

Ah ha!? "Factorization" does not mean that she knew it.

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Sometimes Correct but Not Distinguishing between Card 2 and Card 3

Card 2

- Ahhang could treat letters as numbers in terms of performing substitution, but he did not appear to see the need to justify his “sometimes” answer without probing.
- Even when he performed substitution, he gave up before he completed his calculation.
- When probed, he finished his trial. In short, he appeared to believe that he could test by substitution, yet he was inclined to guess.
- Eventually, he performed one substitution ($a=b=2$, $c=4$) and obtained the answer, “1”, for both sides but he still could not conclude.

Card 3

- Under the influence of card 2.
- He first said that the two statements were the same and needed some help before he could see that the statements on cards 2 and 3 were different statements.
- He then proceeded to substitute $a=4$, $b=c=2$ and concluded the “never” answer.
- However, he later wanted to change to “always” and attempted to verify by putting 3 and 6 into the statement.
- Without finishing his substitution and calculation, he concluded with the “sometimes” answer.

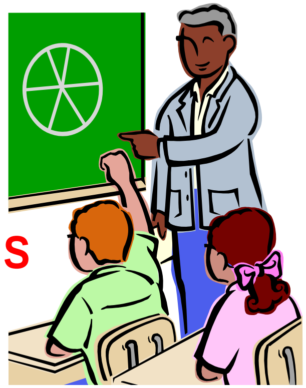
A typical behavior for immature thinking:

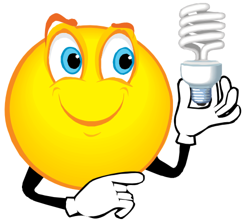
He could be inconsistent and did not see the need for justification.

Rethinking pedagogical issues
重新思考教学问题

- 我们有带领我们的学生站在巨人的肩膀吗？
- Have we brought our students to the shoulder of the giants?

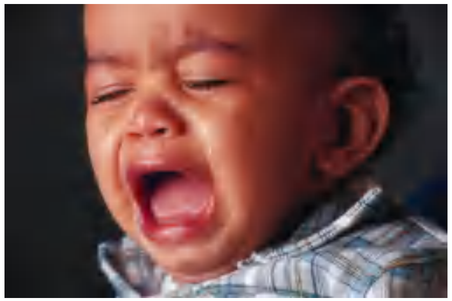
提出问题
raise questions





灵感
inspiration

探索与研究
exploration



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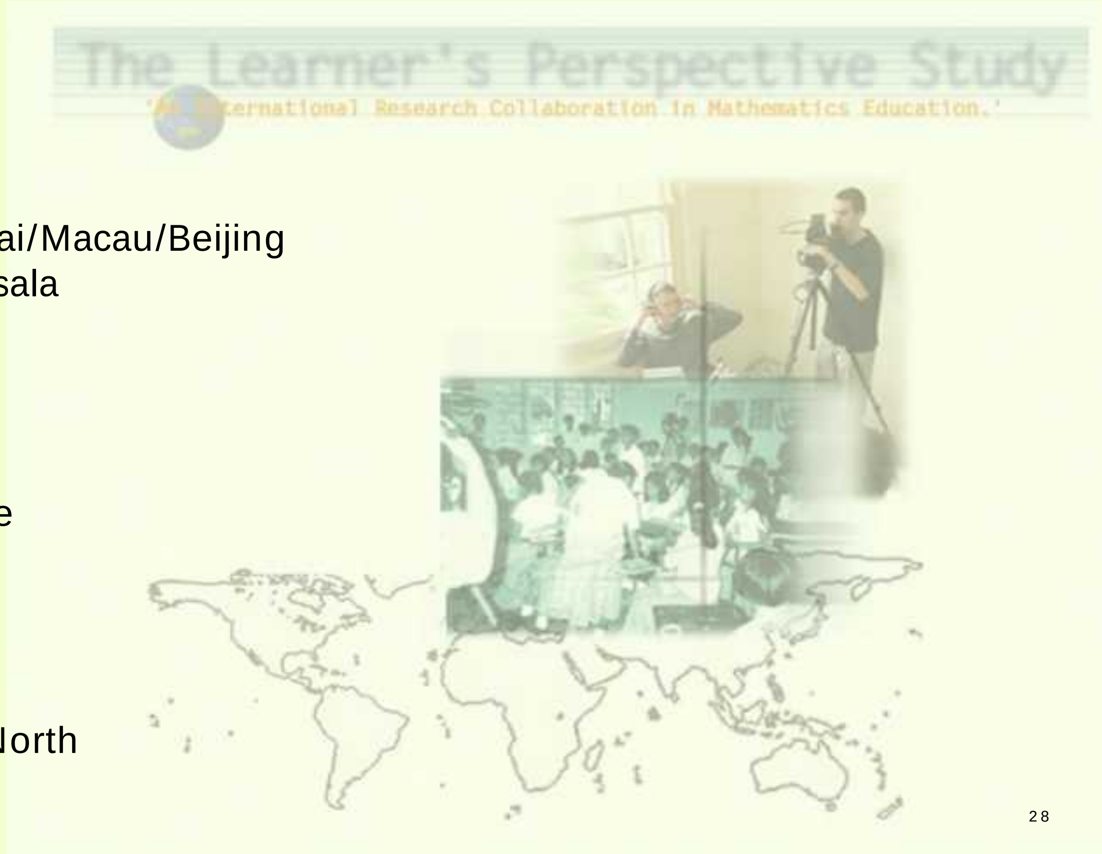
The Learner's Perspective Study

(An international collaboration)

学习者的视角研究
(国际合作）

Learner’s Perspective Study (LPS, since 1999)

1. Australia - Melbourne
2. Germany - Berlin
3. Japan - Tokyo
4. USA - San Diego
5. China - Hong Kong/Shanghai/Macau/Beijing
6. Sweden - Gothenburg/Uppsala
7. South Africa - Durban
8. Israel - Tel Aviv
9. Philippines - Manila
10. Korea - Seoul
11. The Czech Republic - Prague
12. United Kingdom - Bristol
13. Singapore
14. Portugal - Lisbon
15. Norway - Bergen
16. New Zealand - Palmerston North



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- ***Key features in the design***

  - “Record” lessons from multiple perspectives: video, teacher- and student-interviews.
  - Document the teaching of sequences of lessons, rather than single lessons like the TIMSS video study.

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Selection of Sample

- **School Selection:** Schools in urban/metropolitan communities in the two cities (Shanghai and Hong Kong)
- **Teacher Selection:** Three competent teachers in each city (at least five years of experience as a qualified teacher)
- **Class Selection:** One secondary-2 class per teacher, in order to match the database of TIMSS Video Study and the Learner's Perspective Study.
- **Lesson Selection:** A continuous sequence of at least 10 lessons for each class.
- **Content (Mathematics Topic) Selection:**
- **Student Selection:**

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Data Collection

For each country

- eighth grade lessons were recorded in three classrooms (one for each schools).
- a minimum of ten consecutive lessons were recorded for each class/teacher.
  - Camera Configuration**:** a "Teacher Camera", a "Student Camera" and a "Whole Class Camera".
  - **Integrated Video:** the Teacher Camera and Student Camera images in a split-screen arrangement,
  - **Fieldnotes:**
  - **Student Written Work:**
  - **Digitizing of Videos:** VPrism files for the purposes of transcription/translation.

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Interviews and Questionnaires

- **Student** Interviews:
- **Teacher Questionnaires:**
  - (i) preliminary about each teacher's *goals*
  - (ii) very brief *post-lesson* questionnaire; and
  - (iii) reflection on the *lesson sequence*.
- **Teacher** Interviews**:** The data from the questionnaires will be supplemented by three teacher-interviews in which critical issues found during the lessons will be discussed.

Use of research instruments

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Before the starting of videotaping — Teacher Q'ire 1 (once)

Three Weeks of Videotaping
- Teacher Q'ire 2 (everyday)
- Student Interview Protocol (everyday)
- Teacher Interview Protocol (per week)

After the completion of videotaping
- Teacher Q'ire 3 (once)
- Student Maths Test (once)

Data Construction in the LPS

Design elements standardised across 16 participating countries:

- Teacher competence defined by local criteria
- Grade 8 mathematics classes in demographically different urban schools
- Lesson sequences covering one ‘topic’ for each teacher (after a familiarisation period of two or three lessons)
- Video recording of the activities of different pairs of students in each lesson throughout one topic
- Three video cameras (TC, SC, WC) plus on-site mixing to provide interview stimulus
- Post-lesson video-stimulated interviews with students and teacher
- Written materials photocopied and scanned in
- Student test(s) and Teacher questionnaires

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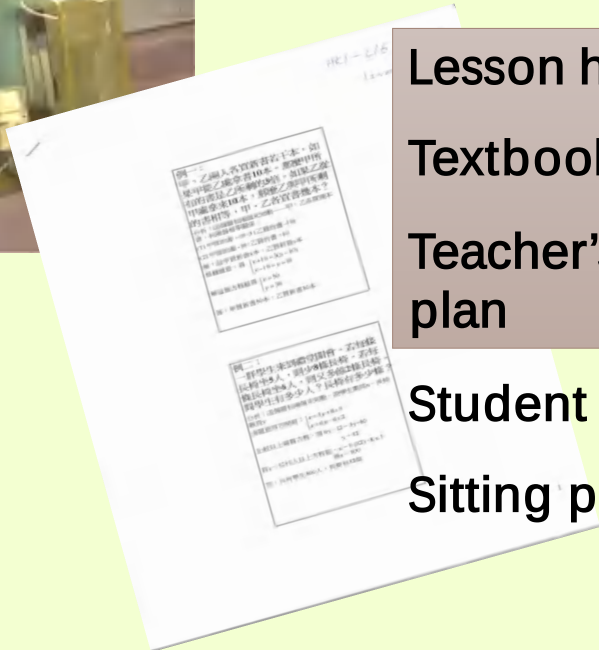
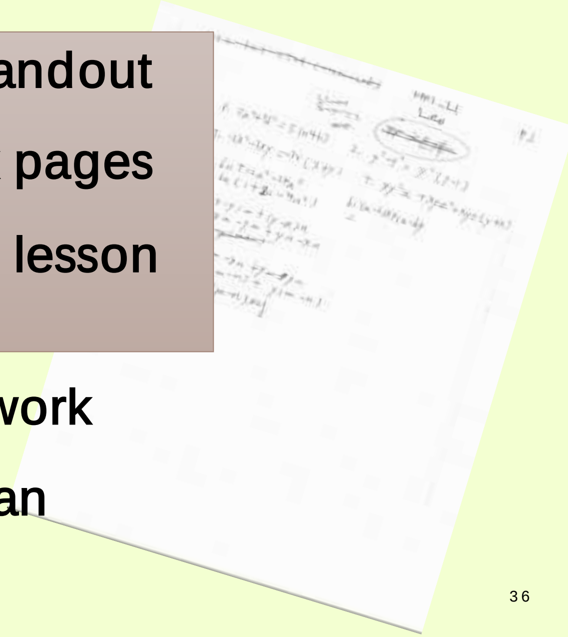
Getting ready

- The schools
- Set up of equipment
- Interviews
- Data
- Analysis





Lesson handout

Textbook pages

Teacher's lesson plan

Student work

Sitting plan

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The Integrated Data Set

- *In relation to a given lesson:*
  - Videotape from Teacher Camera
  - Videotape from Student Camera
  - Videotape of composite Image from Student Camera and Teacher Camera
  - Videotape from Whole Class Camera (The Whole Class Image)
  - Audiotapes of interviews with at least two students
  - Photocopies of written work produced by all four focus students
  - Photocopies of textbook pages, worksheets or other written materials as appropriate
  - Brief post-lesson teacher questionnaire
- *Additional general data set*
  - Student tests (only administered once, after completion of videotaping)
  - Other student achievement data (school data, Internatioal Bench-marking Test)
  - Teacher questionnaire data on teacher goals and beliefs
  - Teacher interview data

The Learner's Perspective Studies (LPS Series)

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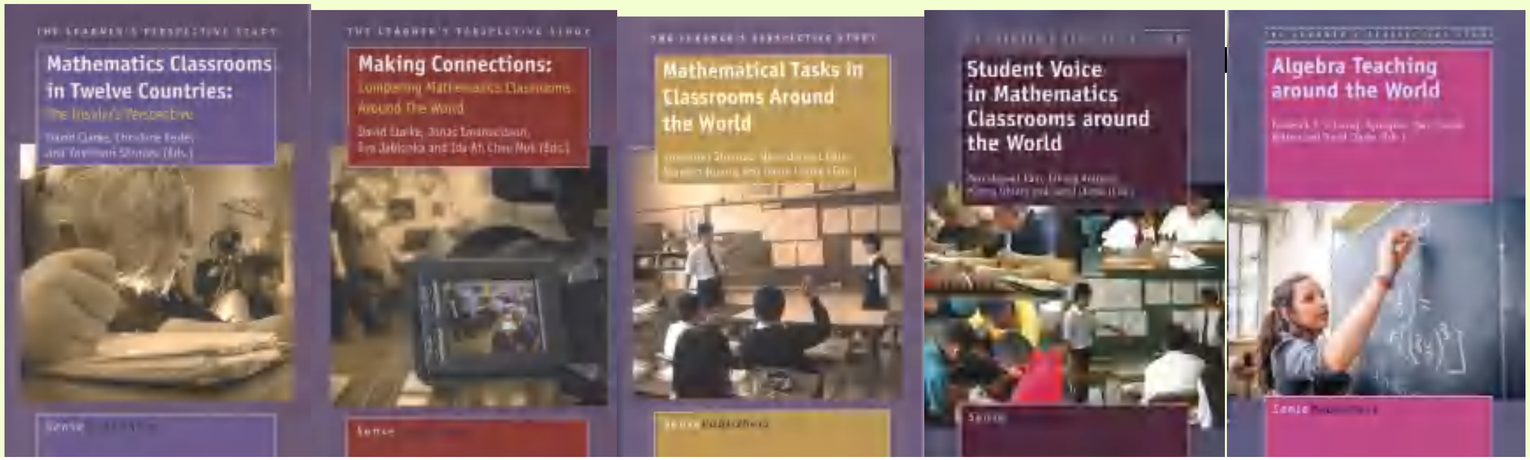
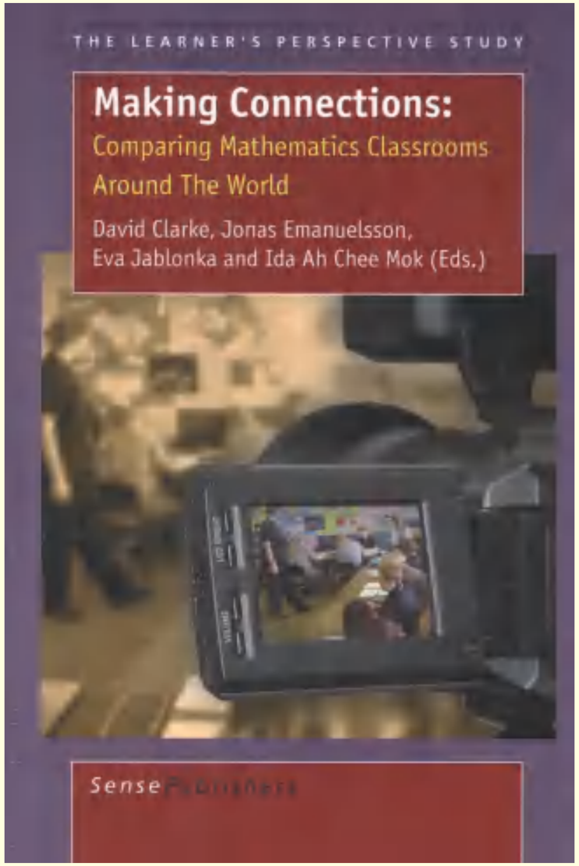
- Clarke, D., Keitel, C. and Shimizu, Y. (Eds.) (2006). Mathematics Classrooms in 12 Countries: The Insiders' Perspective. Rotterdam: Sense Publishers B.V.
- Clarke, D., Emanuelsson, J., Jablonka, E., and Mok, I.A.C. (Eds.) (2006). Making Connections: Comparing Mathematics Classrooms Around the World. Rotterdam: Sense Publishers B.V.
- Shimizu, Y., Kaur B., Huang, R. & Clarke, D. (Eds.). (2010). Mathematical tasks in classrooms around the world. Rotterdam: Sense Publishers B.V.
- Kaur, B., Anthony G., Ohtani, M. & Clarke, D. (Eds.). (2013). Student Voice in Mathematics Classrooms around the World. Rotterdam: Sense Publishers B.V.



Comparing Mathematics Classrooms Around the World



Clarke, D., Emanuelsson, J., Jablonka, E., and Mok, I.A.C. (Eds.) (2006). Making Connections: Comparing Mathematics Classrooms Around the World. Rotterdam: Sense Publishers B.V.

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Lesson Events

- regularity in the form and function of types of the key lesson activities/events from which lessons are constituted, e.g.,
  - Beginning the lesson;
  - Kikan-Shido (between desks instruction);
  - Students at the front;
  - Matome (summary of the lesson); and
  - ‘Learning task’ lesson events

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Beginning of the lesson
(Mesiti and Clarke, 2006)

- First 10 minutes
- USA, Australia, Japan and Sweden.
- The dominant components were:
  - the pre-education component (administrative, organizational, pastoral care);
  - the review component (focusing or warm-up, recap or run-through);
  - the instruction component; the student practice component;
  - the student assessment component (diagnostic, assessment);
  - and the correction component (whole class, independent).

Kikan-Shido / Between desk instruction (O'Keefe, Xu and Clarke, 2006)

- Berlin, Hong Kong, Melbourne, San Diego, Shanghai and Tokyo
- Four mutually exclusive principal functions:
  - monitoring student activity,
  - guiding student activity,
  - organizational action, and
  - social talk.
- Can be purposefully used to distribute the responsibility for knowledge generation in the classrooms of competent teachers within the institutional and cultural norms constraining that practice.

Students at the front: (Jablonka,2006)

- The front of the classroom refers to:
  - the side of the room on which the teacher's desk, the board, an overhead projector (OHP), a flip chart, or a screen was located
- The functions:
  - an extra chance to get the teacher's comments,
  - solving a new task in public,
  - publicizing work, explaining work,
  - providing a division of labour between teacher and students, and
  - displaying work.
- The students' activities:
  - writing solutions on the board, presenting an account of completed work, showing products of group work, or assisting the teacher in a demonstration.

Matome / summing up (Shimizu, 2006)

- The Japanese teachers:
  - teacher public talk, effective use of chalkboard and reference to the textbook; sharing and pulling together the students' solutions in the light of the goals of the lesson of the day. It is important in both teachers' and students' view.
- The Australian teachers:
  - did not give a specific summary at the end of each lesson and they tended to wait until the end of the topic before delivering a summary.
- The German teachers:
  - the teacher did give some summary or provided some general comments on students' procedure, but it did not seem to be common for the German teachers to conclude the lesson by discussing or summarising retrospectively what students had learned during the whole lesson.
- The US teachers:
  - the summary often appeared at the end of each activity, instead of at the end of a lesson.
- Asian classrooms:
  - Japan and Shanghai (similar), Hong Kong (appeared to be different)

Learning Task Lesson Events (Mok and Kaur, 2006)

- A learning task lesson event was defined as comprising not only the description of the task itself but also the actual lesson episode in which the teacher and the students engaged themselves in the task, i.e., both the stated or written task and the subsequent and enfolding social activity.
- Differentiation between a learning task and a practice item.
  - A learning task was intended to teach the students something new and the sequence of learning tasks showed a coherent development for the object of learning,
  - A practice item was mostly repetition of a taught skill.
- They compared 18 learning tasks from Australia, Germany, Hong Kong, Japan, Shanghai, Singapore and United States.

ZDM Mathematics Education (2009) 41:319–332
DOI 10.1007/s11858-009-0166-8

ORIGINAL ARTICLE

In search of an exemplary mathematics lesson in Hong Kong: an algebra lesson on factorization of polynomials

Ida Ah Chee Mok

Accepted: 30 December 2008 / Published online: 22 January 2009

Abstract The author here describes an exemplary grade-8 algebra lesson in Hong Kong, taken from the data of the learners' perspective study. The analysis presents a juxtaposition of the researcher's analysis of the lesson with the teacher and students' perspectives of the lesson. The researcher's perspective applies the theory of variation for which the main concern of learning is the discernment of the key aspects of the object of learning and that the description of variations delineates the potential of the learning space. Some persistent features were illustrated, namely, the teacher talk was a major input in teaching; the technique of variation was used in the design of the mathematical problems and the dimensions of variation created in the class interaction provided a potential learning environment; the teacher taking seriously the student factor into account in his philosophy and practice. From the

of thinking and behavior are found in our daily
words to describe teacher education. For example
teacher education "teacher model" (師範) in
Nevertheless, the search for exemplary lessons in
carries other meanings. Researchers are not look
model to imitate because direct import or tran
foreign model to an existing system seldon
Researchers are well aware that within any
educational system, there are limitations as a
many considerations that may be methodologica
or people's capacity within a culture of values and
practices. The learner perspective study (LPS)
guided by this belief and that we need to learn f
other to get insights into the practices of mat
classrooms in different countries (Clarke et al., 2

The purpose of providing an exemplary lesso

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- Mok, I.A.C. (2009). In search of an exemplary mathematics lesson in Hong Kong: An algebra lesson on factorization of polynomials. *Zentralblatt fuer Didaktik der Mathematik (ZDM Mathematics Education)*. 41, 319-332. DOI 10.1007/s11858-009-0166-8.

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Sung Je Cho
Editor

Selected Regular Lectures from the 12th International Congress on Mathematical Education

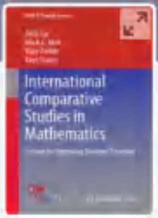
Springer

Research on Mathematics Classroom Practice: An International Perspective

Ida Ah Chee Mok

Abstract Research on Mathematics Classroom Practice encompasses very comprehensive themes and issues that may include any studies and scientific experiments happening inside the classroom, including consideration of the key agents in the classroom (the teachers and the students), undertaken with diversified research objectives and theoretical backgrounds. To a certain extent, seeking an international perspective provides some delineation of the topic. Studies will then focus on those issues already prioritised as of interest by existing international comparative studies and those issues seen as significant within an educational system. This lecture will draw upon the work of an international project, the Learner's Perspective Study (LPS), an international collaboration of 16 countries with the aim of examining in an integrated and comprehensive fashion the patterns of participation in competently taught eighth grade mathematics classrooms.

Keywords Mathematics classroom practice · Cross-cultural practice · Teaching strategies · Learning tasks · Student perspective



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International Comparative Studies in Mathematics

Lessons for Improving Students' Learning

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2.3 *Lesson 3: Changing Classroom Instruction*

This section draws upon the work of two international studies of teaching practice, the TIMSS Video Study and the Learner's Perspective Study (LPS), to zoom into what we may learn from international comparative studies focusing on classroom instruction.

2.3.1 Complementary Roles of TIMSS Video Study and Learner's Perspective Study

The first TIMSS Video Study took place in 1995 (Stigler and Hiebert 1999) and studied national samples of eighth grade mathematics lessons from Germany, Japan, and the USA. The conclusion reported in The Teaching Gap (Stigler and Hiebert 1999) was that teaching was a cultural activity. The design of this TIMSS Video Study combined methodologies of qualitative classroom research and

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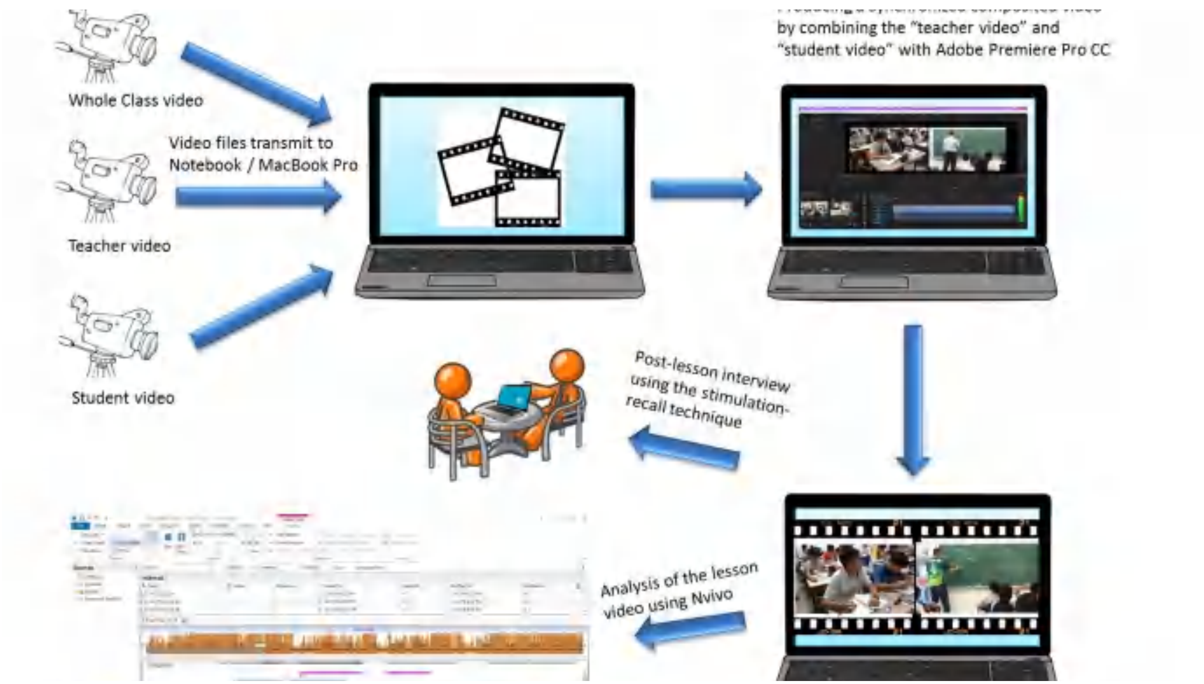
Additional contributions



- Breakthrough in research methodology and technology
- A set of standard procedures in observational classroom research
- Including the students' perspectives in the research design
- Analysis of video data from multiple perspectives

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Integrating Research into Practice: The Growth of Collective Pedagogical Content Knowledge (A collaboration with my doctoral student)

“将研究融入实践：集体教学内容知识(Collective PCK)的进程（与我的博士生合作）

Faculty of Education
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Original Research Article

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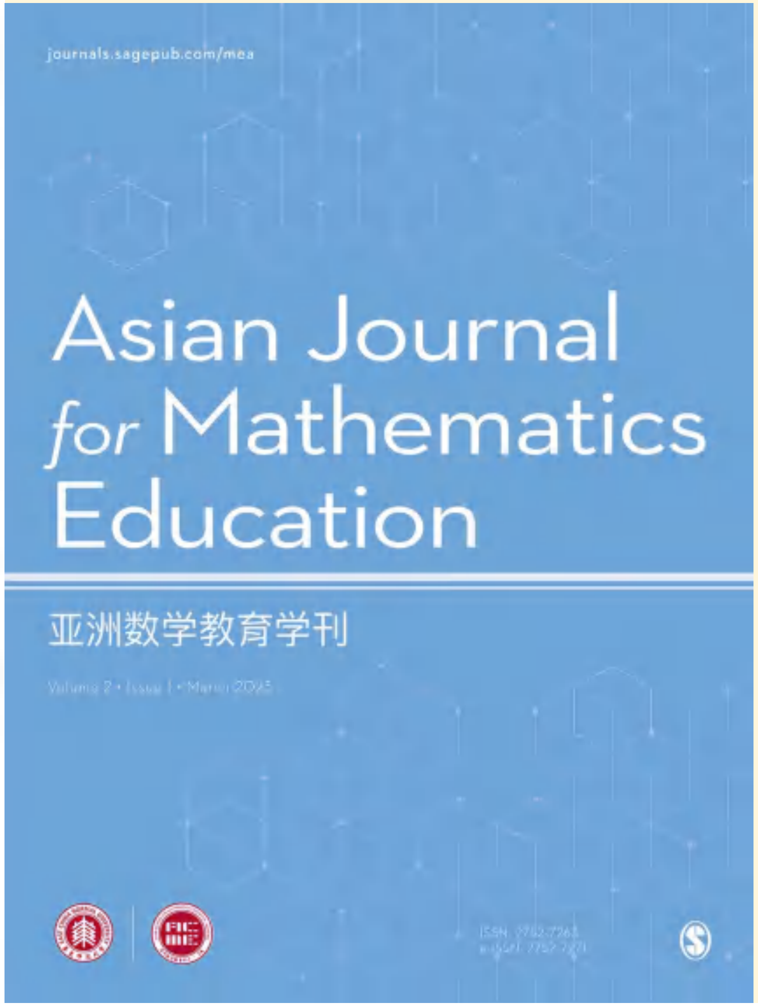
Integrating research into practice: The growth of collective pedagogical content knowledge for primary mathematics via lesson study

Ida Ah Chee Mok[1] and Yee Han Park[2]

Abstract

Teacher is the most important person in enhancing the students' learning and lesson study (LS) has achieved international consensus as a vehicle for developing good lessons and enhancing teachers' professional skills. The aim of this study is to explore how lesson study might act as a form of professional development in helping primary mathematics teachers to develop their collective pedagogical content knowledge in teaching mathematics for the topic of quadrilaterals in the fourth grade. The case study approach with a participant researcher was used. Adapting the sociocultural perspective, the teachers' reflective practice in LS is linked to the teachers' engagement with inquiry and research. The term "*Collective Pedagogical Content Knowledge*" (*collective PCK*) is used to describe the pedagogical content knowledge that is explicitly developed and shared by the LS teachers with evidence identified as "Seed Events" in the analysis. Seed Events are episodes identified in LS teacher meetings or research lessons, leading into deep reflection, consequently clarifying the conceptions of mathematics objects, enhancing the pedagogical strategies, and further enactment in subsequent research lessons. Finally, we further argue that LS is a wise investment of professional capital so that teachers can envision their professional growth with their industrious effort for something worthy.



Dr. Park, Yee Han Miranda:

- 在港大教育学院完成第一个硕士学位(National and Education Development)后，到公开大学申请报读博士学位，因本身是数学老师，想主修数学教育有关的教育博士学位，但因第一个硕士学位并非与数学教育有关，因此不被公开大学取录。于是我再报香港大学的教育学院，报读第二个硕士(Mathematics Education)，完成后，继续报读港大的教育博士学位。由于博士学位主力研究数学教育，与我的硕士学位有密切关系，因此我邀请了当时担任我硕士论文的导师(Dr. Ida Mok)继续担任我的博士论文的导师。这个选择对我来说有极大的好处，因为硕士论文的导师已非常了解我的背景及我的研究兴趣，因此当我与导师讨论博士论文的课题时，很快达成共识。
- 作为小学数学科的科长，我想我的博士论文与数学教师的专业发展有关。因我经常以科长身份参与校内数学老师的集体备课及课研，这正好成为我收集数据的途径。加上当年课研(lesson study)是教育界的热门课题，我的博士论文导师极力鼓励我可以从这方面入手，因此我基于我的研究兴趣、工作提供收集数据的便利、配合教育发展的趋势而选择了我的研究课题。

Key features in the paper

- Lesson Study (LS) acts as a form of professional development
- Collective pedagogical content knowledge (Collective PCK)
- The topic of quadrilaterals in 4th Grade.
- The case-study approach with a participant researcher.
- The socio-cultural perspective: The teachers' reflective practice in LS is linked to the teachers' engagement with inquiry and research.
- The term *"Collective Pedagogical Content Knowledge" (collective PCK)* is used to describe the pedagogical content knowledge that is explicitly developed and shared by the LS teachers with evidences identified as *"Seed Events"* in the analysis.
- Conclusion: LS is a wise investment of professional capitals so that teachers can envision their professional growth with their industrious effort for something worthy.

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Research Questions

The overarching aim for the study is to explore how lesson study might act as a form of professional development in helping primary mathematics teachers to develop their collective pedagogical content knowledge in teaching mathematics. In the investigation, there are two sub-questions:

1. How can the growth of pedagogical content knowledge in a collective manner be interpreted in the process of Lesson Study as a school-based professional development?
2. What factors foster these changes in the Lesson Study?

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Outline of the Paper:

(1) Introduction,

(2) A theoretical background for the construal of lesson study and collective Pedagogical Content Knowledge (collective PCK),

(3) The design of the study,

(4) Results, and

(5) Discussion and Conclusion.

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Lesson Study

- Lesson Study (LS) has become a global trend for enhancing mathematics teaching and teachers' professional development.
- There are different formats of lesson studies and no consensus for the definition of lesson study.
- Huang and Shimizu (2016) carried out a systemic literature review of 52 papers of LS and give a conceptualization of LS.
- Based on this, they summarised 4 types of LS illustrations, namely, Japanese LS, Chinese LS, Learning Study (Sweden/Hong Kong), and UK LS.

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Collective Pedagogical Content Knowledge (PCK)

- According to the seminal work of Shulman (1986), pedagogical content knowledge PCK is of special interest because it is a knowledge that links content and pedagogy.
- The conceptualization of PCK has received much attention and a lot of studies have been carried out for the interpretation of teachers' knowledge in teaching. E.g., Hill, Ball, and Schilling (2008) ; Jaworski and Huang (2014); Kieran, et al., (2012).
- Through the collaborative planning and examination of actual lessons, teachers' subject matter knowledge, knowledge of students understanding, curriculum knowledge, and knowledge of instructional strategies are involved (Cajkler, et al. 2014; Huang and Shimizu, 2016; Kieran, et al., 2012).
- The term "Collective Pedagogical Content Knowledge" (CPCK) is created to describe the pedagogical content knowledge that is explicitly developed and shared by the teachers from lesson study group with evidence provided in their pre-lesson preparation meetings as well as post-lesson evaluation meetings, supplemented with their peer-observed research lessons.

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The collective PCK framework

- The collective PCK framework applied in the analysis of data consists four components, namely, subject matter knowledge, knowledge of students' understanding, curriculum knowledge, and knowledge of instructional strategies, and their interconnections.
- The four components in collective PCK and PCK are the same.
- A major difference between CPCK and PCK is where the evidences are found. In many studies, PCK refers to that found in individual teacher, and may be probed with research instruments (e.g., Ma, 1999). Collective PCK in this study refers to the knowledge generated in the discourse between the teachers in the preparation and evaluation of research lessons in the process of LS.

The Case School and Participant Teachers

- The study has adopted a qualitative case study approach to explore the experiences of a group of 5 teachers engaged in one lesson study group.
- The case school was an established school with a good reputation in its local district, for its strong emphasis on mathematics and student performance in mathematics is generally higher than average for Hong Kong. The school was a bi-sessional school with teachers of the am school and pm schools working closely in collaborative lesson planning (or lesson study).
- The school principal was an innovative leader, who encouraged school reforms, and lesson study was utilised as a research tool for enhancing the teacher professional development in the school.
- Research Ethics Approval: The school-based staff development project of lesson study was led by the mathematics panel chairperson and supported by the school principal and colleagues. At the same time, the panel chairperson got consent to use the data generated in the process for her own study for her doctoral degree thesis.

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The LS team

- The panel chairperson, playing the role of a participant researcher and the coordinator leading the Lesson Study group.
- For the topic of the research lesson was for primary 4, only the teachers teaching the primary 4 classes were included in this LS group.
- The teachers were helpful, industrious, and willing to share their experiences with others. This made it feasible to detect and illustrate developments in their pedagogical content knowledge and the subsequent effects on their teaching practices and the experiences of the students.

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The procedure of the lesson study are listed below in chronological order according to the school timetable:

4 Pre-lesson meetings to develop a lesson plan

1. Research Lesson 1 (Class 4A pm school, Teacher G) & Post-lesson meeting 1
2. Research Lesson 2 (Class 4B am school, Teacher F) & Post-lesson meeting 2
3. Research Lesson 3 (Class 4B pm school, Teacher C) & Post-lesson meeting 3
4. Research Lesson 4 (Class 4A am school, Teacher E) & Post-lesson meeting 4

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A typical lesson study process consists of 6 steps:

(i) collaboratively planning the research lesson,

(ii) observing implementation of the research lesson,

(iii) discussing the study lesson,

(iv) revising the lesson plan (optional),

(v) teaching the new version of the lesson (optional), and

(vi) sharing reflections about the new version of the lesson (Fernandez & Yoshida, 2004 ; Kieran, et al., 2012).

- In addition, teachers will have the opportunity to develop a strong pedagogical content knowledge with their colleagues through lesson study.

Analysis

- Coding of the Pre-lesson, Post-lesson Meetings and the Research lessons
- Seed Events

Coding

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Table 2 Categories for classifying pedagogical content knowledge

Categories	Sub- Categories	Examples
Subject matter knowledge	• Properties of quadrilaterals • Definitions of quadrilaterals • Sub-class relationships among quadrilaterals	• The definition of trapezium • The relation between a rhombus and a square
Curriculum knowledge	• Sequence and logic of teaching content • Vertical curriculum development across grade levels • Horizontal curriculum development in the same topic	• The teaching of parallel and perpendicular before the classification of quadrilaterals
Knowledge of student understanding	• Student misconception • underlying reasons for misconceptions • Pre-requisite knowledge the students will already possess • Weakness and strength of the students • Abilities of the students • Difficulties students may have learning the subject	• Opposite sides vs. parallel lines • The definition of parallel • The concepts of parallel and perpendicular
Knowledge of teaching strategies	• Higher level learning approach • The use of demonstrations • The use of examples • The use of explanation • The use of teaching aids	• The flow of the lesson • Using metaphor • Using plastic sticks

Table 3 An example of the open coding approach in the analysis of the meeting transcripts

Post-lesson Meeting 1 Teacher F gave her reason why students only marked one pair opposite sides of a trapezium. "Because of the concept of parallel lines, students thought that a right-angled trapezium only had one pair of opposite sides with the "top and bottom" sides while the "left and right" could not be defined as opposite sides because they were not in parallel. The result showed that students considered the opposite sides and parallel sides of a quadrilateral were the same things"
Category: subject matter knowledge, knowledge of student understanding
Coding notes: observation of students, properties of trapezium, student misconception (opposite sides vs. parallel lines) , the underlying reasons for student misconception (the concept of parallel lines)
Sub-coding: subject matter knowledge: properties of quadrilaterals knowledge of student understanding: student misconception/ underlying reasons for student misconception

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Seed events

Table 4 A summary of seed events

Seed Event	Sources	Characteristics	No. of follow-up actions
A	*RL 1	Student misconceptions (The mix up of opposite sides and parallel lines of quadrilaterals)	3
B	*PLM 1	Teaching strategies (The sequence and logic of teaching content)	1
C	PLM 1	Teacher misconceptions (The definition of trapezium)	1
D	PLM 1 PLM 2	Teacher's own doubt about mathematical concept (The relationship between a rhombus and a square)	1
E	PLM 2 PLM 3	Teacher's fear (Teacher unwilling to teach the sub-class relationship among different kinds of quadrilaterals)	2
F	RL 1 PLM 1	Student misconceptions (The misunderstanding of the concept of parallel)	1
G	PLM 2 PLM 3	Teaching strategies (The use of teaching aids)	4

*RL: Research Lesson. PLM: Pre-Lesson Meeting

No. of follow-up actions: After the discussion about the method or strategy to handle the problem from the seed event, the follow-up actions might be carried out in the following research lessons for the improvement.

Dr. Park: 研究旅程中有哪些关键时刻？

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- 选择研究课题: 论文的研究课题是博士学位最重要的一环，这个关键时刻应该追溯至我的第二个硕士论文的研究课题的选择。当年我打算以「The use of calculator」作为我的硕士论文的课题，当我递表申请的时候，我的论文导师认为这个课题已不合时宜，亦发挥不大，建议我改以「PCK」作为我的研究主题。我二话不说接纳了导师的提议，从新开始研读这方面的学术文章。结果以「Another Topic」完成我的硕士论文。正因为这个改变，我的博士论文亦以「PCK」为主线，加入「lesson study」，发展为我的研究范畴。
- 收集数据: 数据收集极为重要，由于数据来自任教的学校，在校内进行录影、访问、问卷调查均提供一定程度上的方便。
- 分析数据及撰写研究结果: 进入分析数据阶段，是我PhD旅程中最难忘的。因为在整理数据时，好一段日子没有突破性的意念，令自己跌入深渊中，写文进度几乎处于冰封状态。我因为写文的动力太弱，结果需要多次申请 extension。其实论文导师在这期间扮演亦师亦友的角色，一方面不断给我新的意见，让我在分析数据能够突破我的盲点。另方面她亦经常给我劝导，给我提醒，把我从深渊中拉上来。感恩导师对我的不离不弃与无限支援，让我没有中途放弃，直至完成。

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Discussion and Conclusion

- The study specifically look into the development of collective PCK shared in the LS team discourse in the research lessons and LS teacher meetings. In the analysis, “seed events” were identified in LS teacher meetings or research lessons. Seed events were episodes, in which students’ misconceptions or the teachers’ shortcomings in teaching were discovered, leading the LS team into deep reflection, consequently clarifying the conceptions of mathematics objects, enhancing the pedagogical strategies, and further development in subsequent research lessons. Thus, the dynamics in the “seed events” gave evidence for the growth of collective PCK in the LS community.

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What possible factors may contributed to the development of collective pedagogical knowledge in the seed events?

The emergent factors can be summarised in three themes:

- alignment with the school goal of staff development via LS,
- changing from the traditional leader-follower norm to collaborative team professional development, and
- growth of collective PCK in the direction for enhancing teachers' enquiry capacity.

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Finally,

Taking the stance of a socio-cultural perspective, this case study has presented evidences of the teachers' growth of collective PCK via the dynamics in the "seed events" in the process of LS.

While lesson study helps integrating research into practice, warranting the feasibility of teachers becoming action researchers within a sharing culture inside the school (Huang and Shimizu, 2016; Jaworski and Huang, 2014; Kieran, et al., 2012; White, et al., 2012), **teachers can envision their professional growth with their industrious effort for something worthy.**

谢谢
Thank you

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