



Rethinking mathematics teachers' resources in a time of digitalization: towards new programs of research

数字化时代数学教师资源的再思考：新研究项目前景与展望



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Lecture at the Asian Center of Mathematics Education, ECNU  
Shanghai, November 21th

# A French-Chinese collaboration

Thank you Lianghuo for your invitation

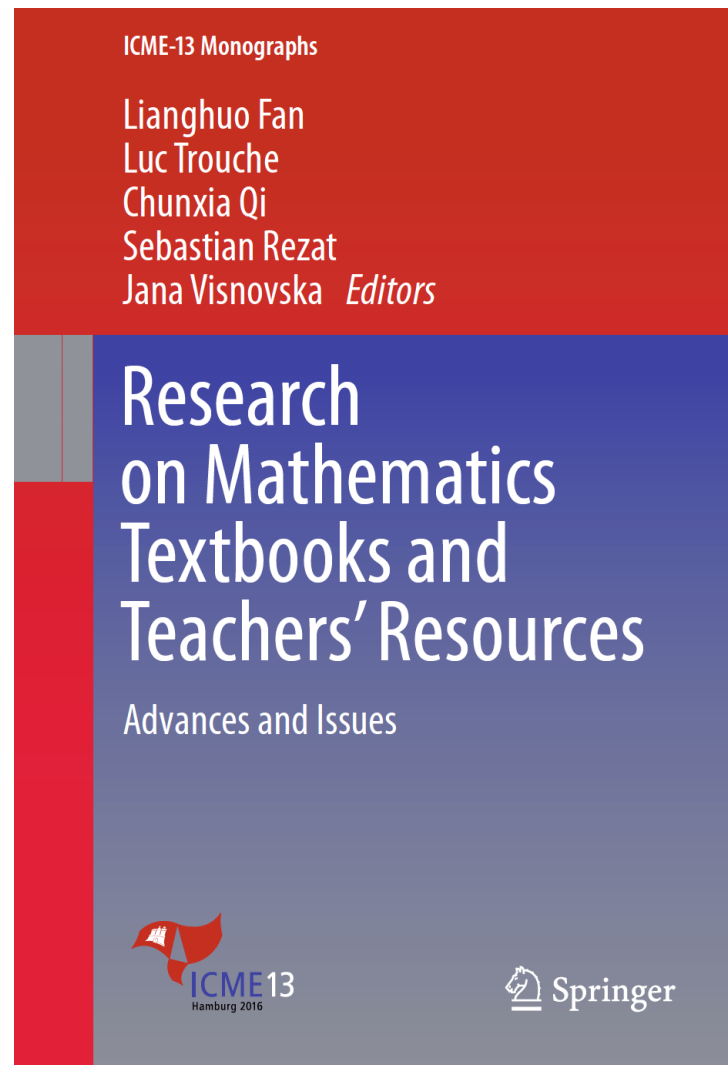
A recent, but intense collaboration

Co-chairing of the Topic Study Group *Research on mathematics teachers' resources (textbooks, learning materials, etc.)* at the International Congress on Mathematical Education (ICME-13) Hamburg 2016

A collaboration resulting in a book

The promising notion of “resources”, beyond the textbooks

A pleasure to visit the Asian Center on Mathematics Education



# A French-Chinese collaboration

After ICME-13, 2016 in Hambourg... ICME-14 to be held in 2020 in Shanghai, hosted by ECNU

I am part of its International Program Committee (IPC)

- Pr Wang Jianpan, chair of the IPC
- Pr Xu Binyan and Pr Bao Jiansheng, co-chairs of the organizing committee

Very happy and honored to work with them for preparing ICME-14



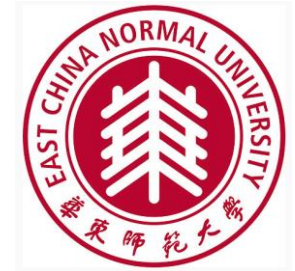
# A French-Chinese collaboration

With Pr Xu Binyan and Pr Bao Jiansheng, an ancient collaboration, in Mathematics Education

Two joint programs ECNU – French “Ecoles normales”:

- JoRISS, a Joint Research Initiative for Science and Society
- ProSfer: pre-doctoral teaching program

I am currently working in ECNU, during two weeks with 8 master students, aiming to do a PhD in France and China



# A French-Chinese collaboration



Chongyang WANG



Binyan XU



Luc TROUCHE



Jiansheng BAO



Luxizi ZHANG



Fangchun ZHU



Sophie SOURY-LAVERGNE



Jana TRGALOVA



Mingyu SHAO

Thank you Pr Xu for the picture!

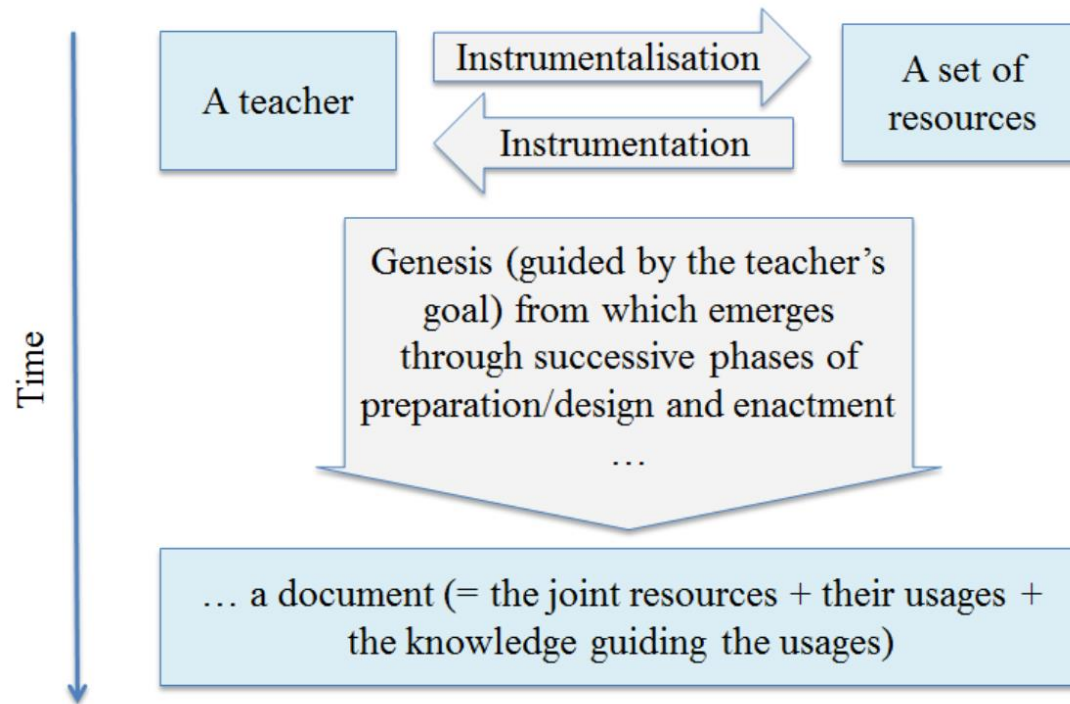
Four PhD currently co-supervised. Sharing a same interest for the mathematics teachers' resources, from different points of view...

## Outlines

1. The emerging of a new theoretical frame for thinking the mathematics teachers' resources in a time of digitalization
2. Ten years after its emergence, evidencing ten missing resources of this approach
3. Understanding the conditions / effects of teachers' collective documentation work
4. A French case study
5. Some perspectives



# 1. The emerging of a new theoretical frame for thinking the mathematics teachers' resources in a time of digitalization



A reflection linked to the digitalization of information, leading teachers to use resources far from only textbooks (Pepin, Gueudet & Trouche 2017)

The development, ten years ago, of the *documentational approach to didactics* (DAD), the critical notion of *teachers' resource systems* (Gueudet & Trouche 2009; Trouche, Gueudet & Trouche 2018)

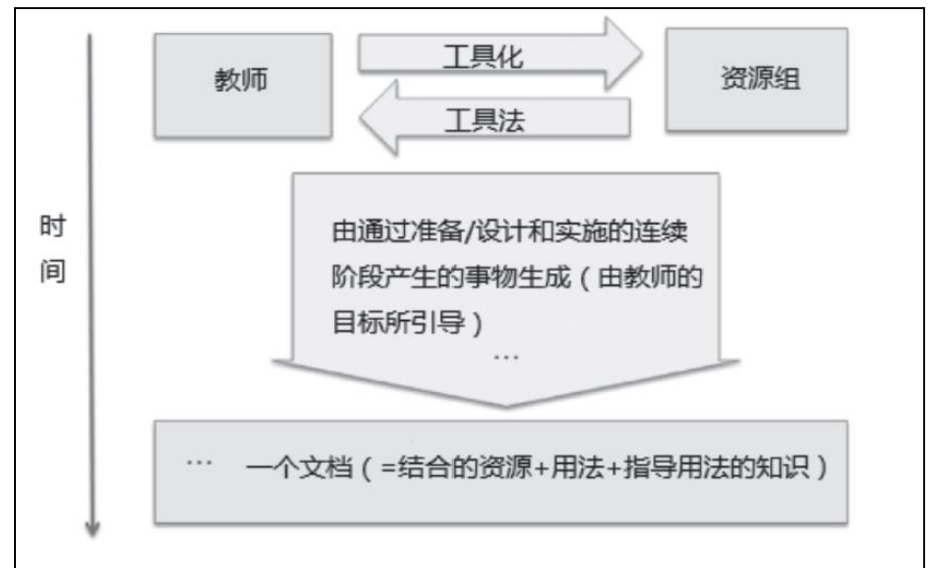
# 1. The emerging of a new theoretical frame for thinking the mathematics teachers' resources in a time of digitalization

This theoretical frame has disseminated in different countries, thanks to PhD co-supervised, international project of research (JoRISS), and conferences

I discovered, in my seminar in ECNU (last week, old campus), a paper proposing, in Chinese, a first view of this approach (Qiao 2018)

A fruitful reflection to be continued, as it is not so easy to translate, terms by terms, some concepts which have not an “exact” equivalent in English (or French), and Chinese

A reflection leading to deepening the concepts at stake



(Qiao, X, 2018)



	Instrumentation	Instrumentalisation	
Translation issues	QIAO Xinhong	工具法	工具化
	SHAO Mingyu	工具应用	工具化

I asked the PhD students, currently in Lyon, for comments. Mingyu (first year) proposed a new perspective in adding to each translation an explanation, opening then a window on Chinese meanings

“For instrumentation, 工具法 just means 工具 (instrument or artifact) + 法 (method, i.e. *instrument method* or *the method of instrument*). In Chinese that demonstrates a very ambiguous relationship between *method* and *instrument*. An instrument itself could constitute one kind of *method for solving math problems* (but in fact there could be many different instruments for accomplish different tasks); or a *research method that relies highly on instruments* to approach teaching/learning resources... That is why I feel 工具应用 (*the utilization of instruments*) would be better and avoid the "method".

More globally, many man-made terms in math education don't give much signification by themselves unless they are followed by more explicit explanation. 工具化 and 工具法, in themselves, would indeed make the Chinese confused. I suggest then:

- 工具化 for instrumentalisation, followed by the explanation 人对工具的个性化改造 (the subject's personalized modification and creation onto the artifact);
- 工具应用 for instrumentation, followed by the explanation 工具对人使用习惯的影响 (influence of the artifact on the subject's habit of utilization).”

# 1. The emerging of a new theoretical frame for thinking the mathematics teachers' resources in a time of digitalization

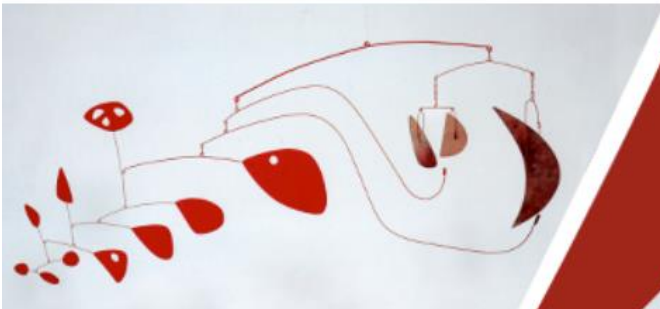
What happened over 10 years of development?

It is to take stock of the advances and issues that a three-day conference has been organized in France in May 2018

It gathered 130 people from 30 countries (including many PhD students)



<https://resources-2018.sciencesconf.org/>



**RE(S)OURCES 2018  
INTERNATIONAL CONFERENCE**

UNDERSTANDING TEACHERS' WORK THROUGH  
THEIR INTERACTIONS WITH RESOURCES FOR TEACHING

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## 2. Ten years after the emergence of DAD, evidencing some missing, theoretical as well as methodological, resources

The development of collaborations, at a national or international level, involving Brazil, China, Japan, Lebanon, Mexico, Netherlands, Senegal, UK and the US, had already allowed to question some aspects of this theoretical frame.

The international conference Re(s)ources 2018 was a good occasion for this reflective thinking

And the point of view of PhD students, having developed this approach in appropriating it, was strongly taken into account.



## 2. Ten years after the emergence of DAD, evidencing some missing, theoretical as well as methodological, resources

Three questions asked to (current or past) PhD and post-doc students:

- a. You have used instrumental approach (IAD) vs. documental approach to didactics (DAD). To which extent did these frames support your research?
- b. In using DAD, did you feel some missing resources? In which circumstances? What new resources (theoretical as well as methodological) this approach should develop?
- c. For designing missing resources, which possible theoretical crossings? Which new research programs to be launched?

29 answers, to be combined with the students' real work and their contributions to the field



## 2. Ten years after the emergence of DAD, evidencing some missing, theoretical as well as methodological, resources

The analysis of the questionnaires leads to the proposition of 10 programs, as possible path to draw vs. to explore (Trouche 2018)

Program 1: Designing a living multi-language glossary of the approach

Program 2: Deepening the structure of a teacher's resource system(s)

Program 3: Deepening the dialectics of schemes / situations of work

Program 4: Deepening the conditions / effects of teachers collective documentation work

Program 5: Understanding the short vs. long episodes of teachers' documentation work

Program 6: Stimulating reflectivity; storing and analyzing related data

Program 7: Developing tools for analyzing the big data related to teachers interacting with resources

Program 8: Thinking specific supports for capturing data related to teachers' documentation work

Program 9: Thinking theoretical networking

Program 10: Studying *naming systems* used by teachers to describe their documentation work

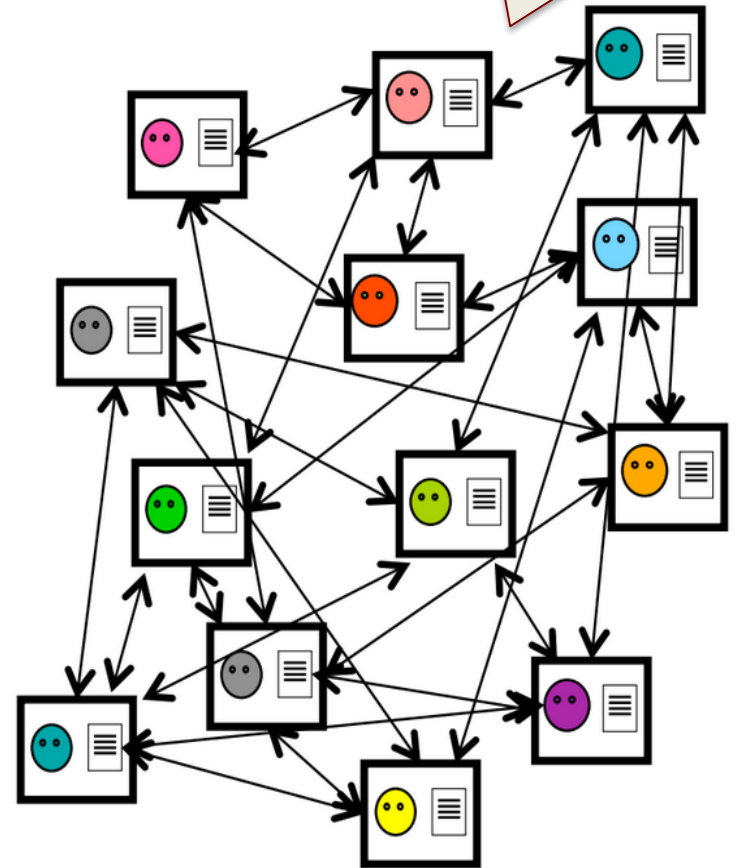
### 3. Understanding the conditions / effects of teachers' collective work

The point of view of teacher's *resource system* as a very structured set of "things" (re)sourcing his/her work, far from being limited to textbooks, in a time of digitalization

Studying the interactions between individual and collective teachers' resource systems, a very complex issue

The institutional place of teachers' collective work in China (Teaching Research Groups, Lesson Preparation Groups), and the development of collective of volunteers teachers in France, a stimulating frame for contrasting case studies (Wang 2018)

We miss tools for dynamically following up the documentation work of a given community

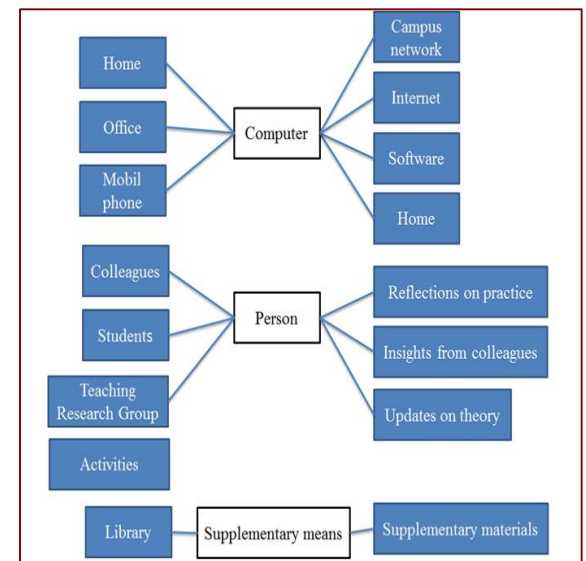
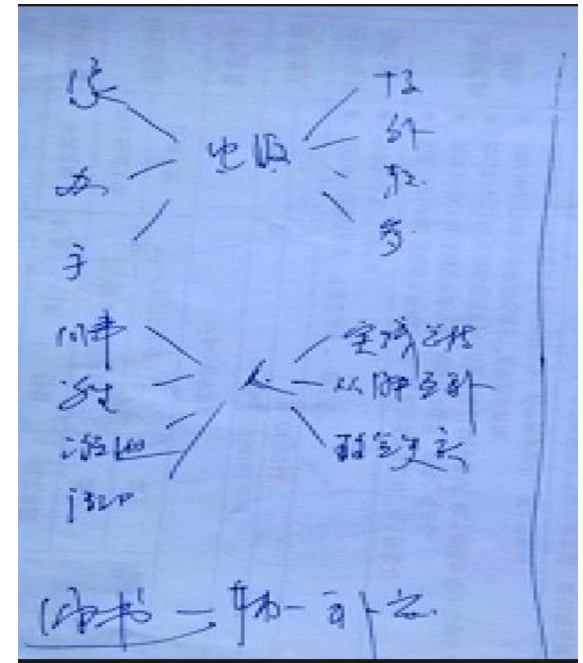


### 3. Understanding the conditions / effects of teachers' collective work

*A first perspective, a static one:*

- Analyzing the individual teachers' resource system, through the eyes of the researcher...
- ... and through the eyes of the teacher him/herself: the fruitful instrument of *Reflective Mappings of Teachers' Resource Systems*
- Crossing individual mappings for understanding the effects of teachers' collective work

As an example: In China, crossing and contrasting the reflective mapping of three expert teachers, of different experiences, see left, from the work of Pepin, Xu, Trouche & Wang (2016)



### 3. Understanding the conditions / effects of teachers' collective work

*A second perspective, a dynamic one:*

- Analyzing the structure of episodes of teachers' real documentation work, when working together for facing a given issue;
- Choosing carefully a minimal collective (2 teachers) with a high probability of strong interactions: a pair of "documentation working mates" (Wang 2018)
- Choosing the preparation of a lesson in a time of deep (curricular) change, where teachers have really: to mutually question the way to introduce the knowledge to be taught; to look for new resources in this perspective; to re-arrange the ancient ones..

The productive setting of *documentation-working mates* (Wang 2018)



Anna et Cindy preparing a new lesson on algorithmics for a 7<sup>th</sup> grade class



## 4. A French case study



Ana and Cindy, two advanced teachers working closely together, in their middle school; and out of their school, mainly in Sesames, a working group with researchers, aiming to renew the teaching of Algebra, conceived as *a program of computation*

In 2016, they had to face a new curriculum introducing programming in math teaching, and the use of dedicated software as Scratch.

We followed their first working session for planning their teaching of programming (Trouche, Gitirana, Miyakawa & Wang Online First 2018).

They are in their usual place, at school, having in front of them the set of available textbook (about 15 different ones, to be chosen by the school), their laptops, and smartphones, for visiting a lot of possible resources.



Anna et Cindy preparing a new lesson on algorithmics for a 7<sup>th</sup> grade class

## 4. A French case study



Textbooks...

Wide web...



The school math teachers store their resources on the school website, in individual or shared folder

0- Resources for Interactive whiteboard	22/02/2015 15:48
Grade 9 lesson plans	10/03/2016 18:17
Grade 9 - Tom	09/09/2016 18:18
Grade 9 - Cindy	22/03/2016 17:26
Grade 9 - Jim	25/04/2016 14:48
Grade 9 - Anna	26/09/2015 14:41
Grade 8 - Cindy	26/09/2015 14:41
Grade 8 - Anna	26/09/2015 14:41
Grade 7 - Tom	26/09/2015 14:41
Grade 7 - Cindy	26/09/2015 14:41
Grade 7 - Anna	06/04/2016 19:50
Grade 6 - Tom	28/08/2016 09:21
Grade 6 - Cindy	25/05/2016 21:07
Grade 6 - Anna	25/05/2016 21:15
Research activities - John	28/08/2016 09:21
Cycle 4 - Algorithmics	10/09/2016 15:19
Mental computation	29/11/2013 18:54
Grade 9 - Alex	26/09/2015 14:41
Reports on students with special needs	27/01/2014 20:27
Activities for various grades	29/11/2013 20:04
Examples of talks	30/11/2013 00:39
Geometry for all grades	22/10/2014 19:31
Games and math narratives	22/07/2014 07:51

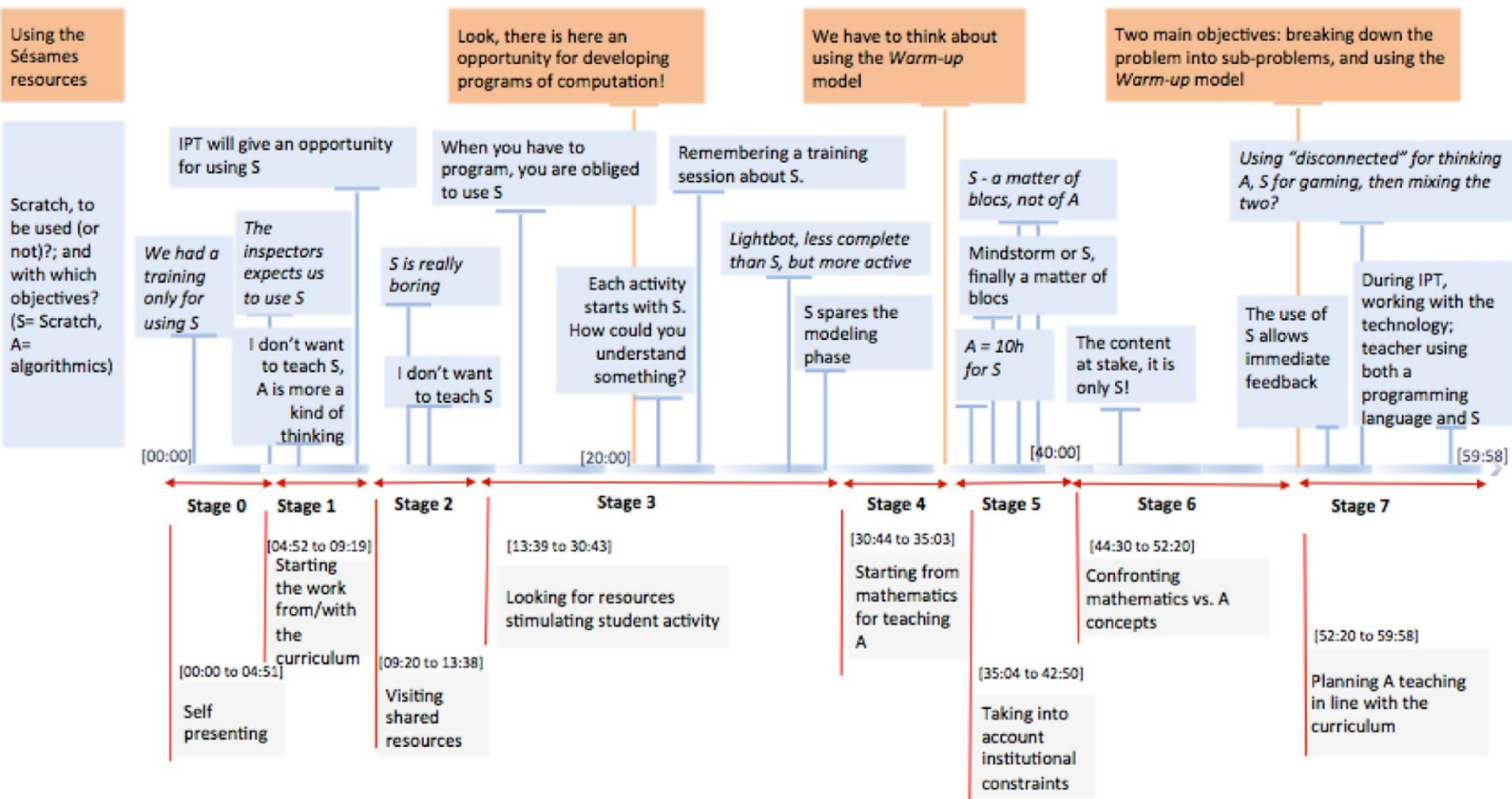
Middle school resource system

Sésames website stores resources structured on a given model, “warming up”, produced of the team experience over the time



Sésames resource system

# 4. A French case study



Lexicon: IPT = Interdisciplinary Practical teaching

Font: normal, Anna; italics, Cindy

Analyzing the dynamics of the work of Anna et Cindy looking for resources for facing a given issue through different stages

## The structure of teachers' work

Stage 1: analyzing the curriculum

Stage 2: visiting shared resources

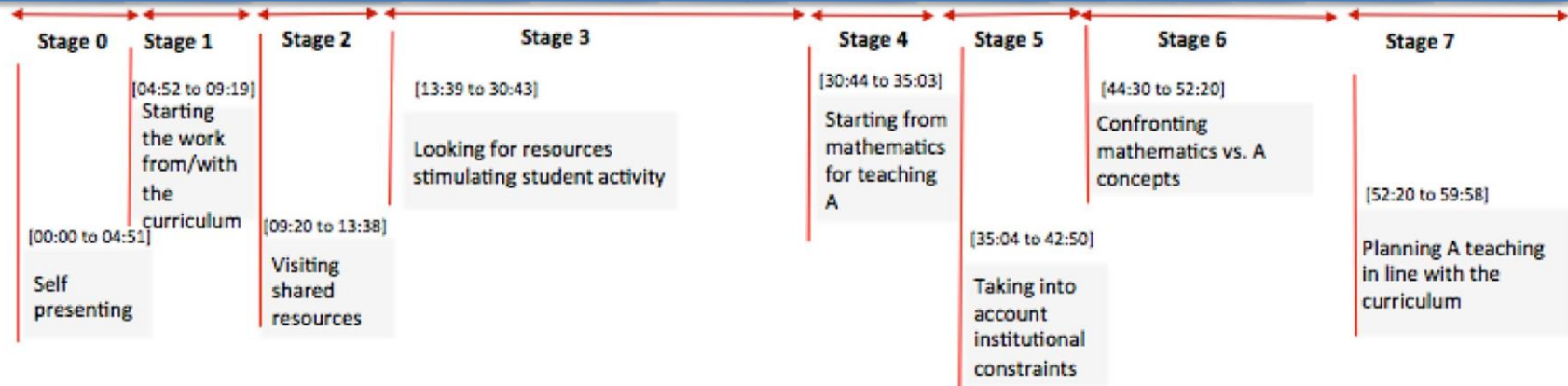
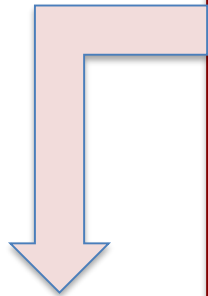
Stage 3: selecting resources for stimulating students' activity

Stage 4: trying to integrate algorithmics in the mathematics progression

Stage 5: taking into account institutional constraints

Stage 6: confronting mathematics and algorithmics concepts (example: the notion of *variable*)

...

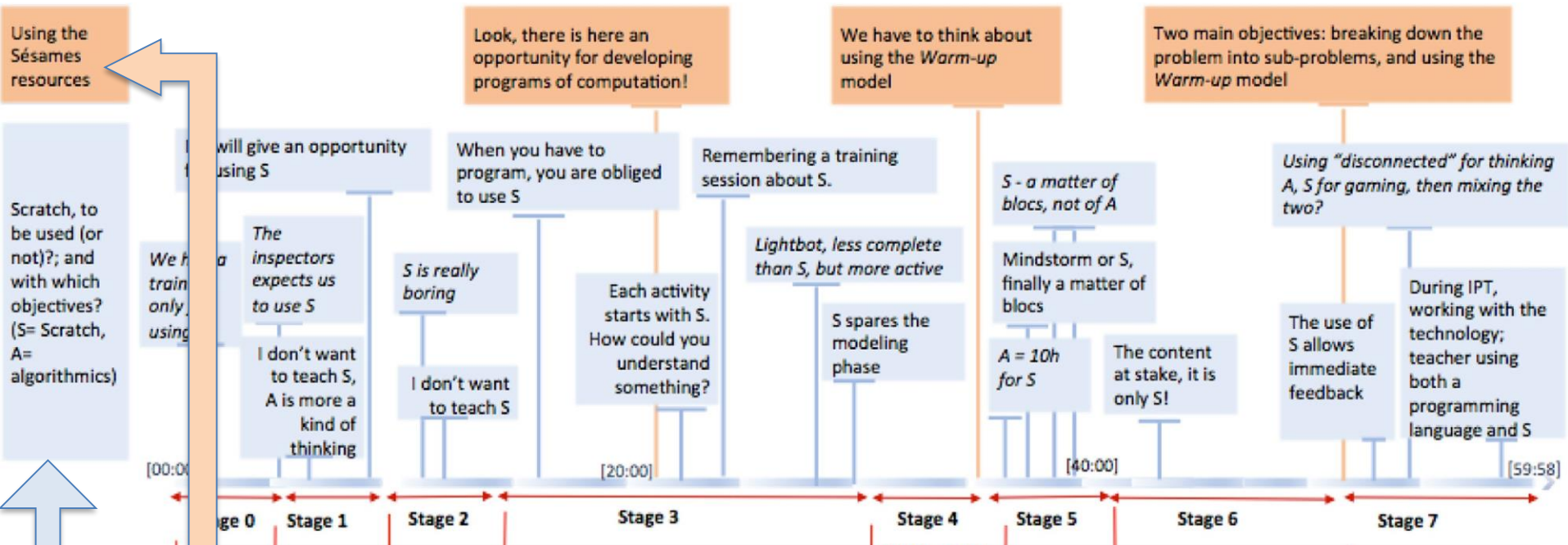


Lexicon: IPT = Interdisciplinary Practical teaching

Font: normal, Anna; italics, Cindy

The development of the two teachers' activity over successive stages

# Nature of the interactions between the two teachers



One main re-source, "warming-up", from the Sésames team

- As a model, it provides a guide for designing new resources
- Structuring classroom, it allows to integrate the new within the usual
- It allows to conceive programming as follow-up of programs of computation

One main issue: Scratch, to be used, or not, an for which purpose?

- "my role, as a math teacher, is not to 'teach a software'"
- "I prefer to use unplugged computing activities"

## 4. A French case study

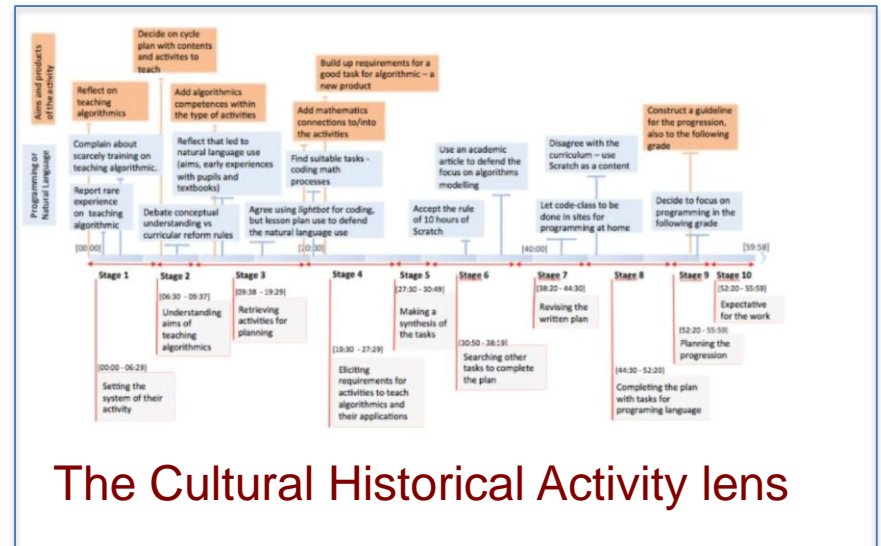
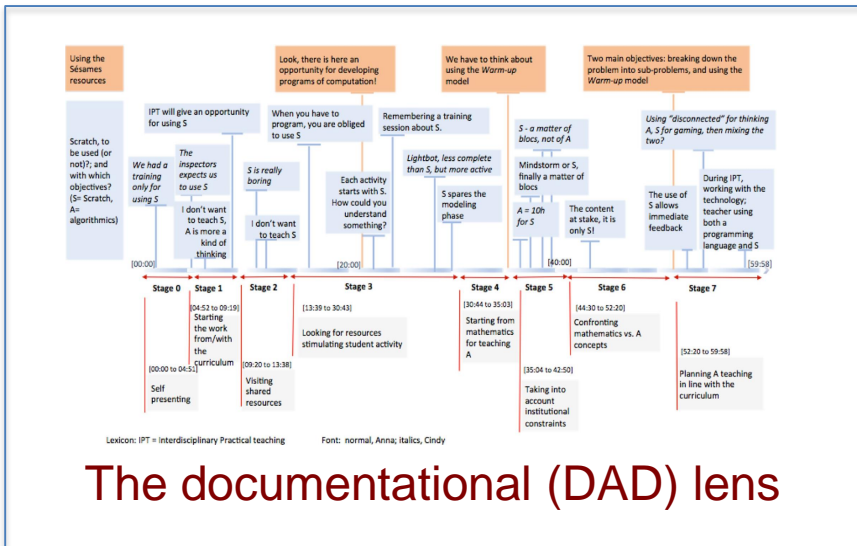
The collective work develops from their familiar shared resources (school resource system).

Interacting for facing the issue of teaching a new topic leads them to a mathematical, didactical, epistemological, curricular, bottom-up reflection

Needing the support of *permeating resources* (in this case, “Warming up”) for insuring the transition from well known to new topics to be taught

Designing permeating resources: a feature of advanced collective (in this case, Sésames) and advanced teachers?

The complexity of collective teacher work, even for two teachers, analyzed in Trouche et al. (Online First) over three different theoretical perspectives



The documental (DAD) lens

The Cultural Historical Activity lens

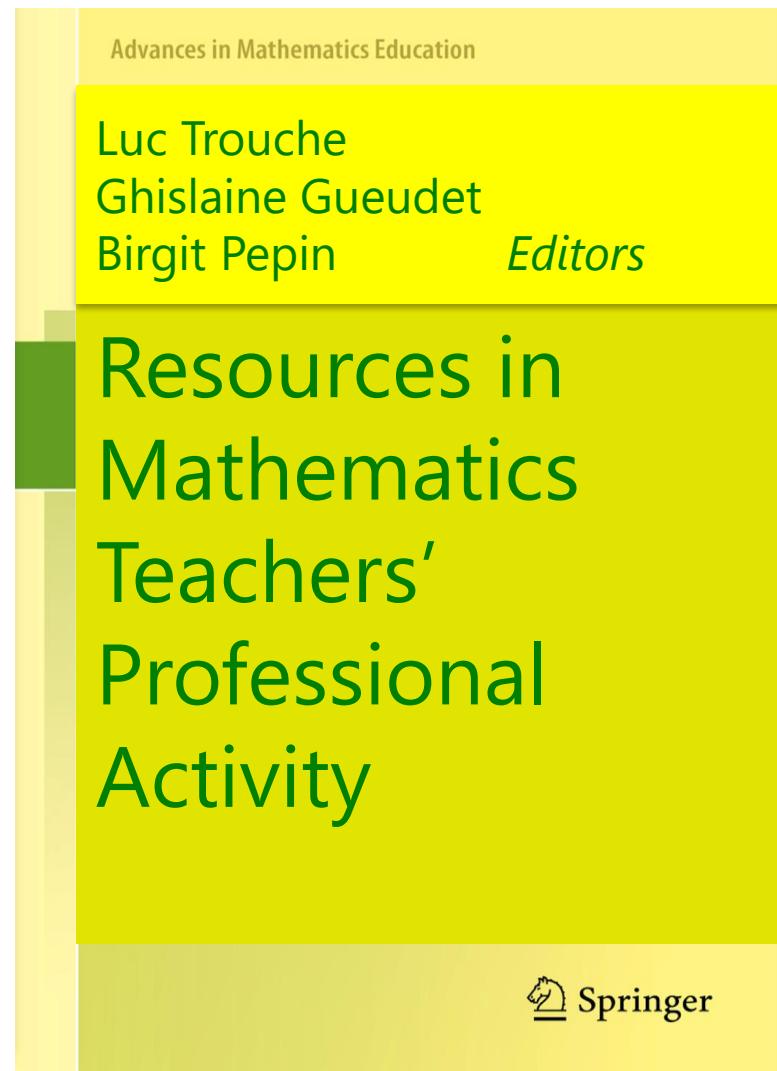
## 4. Perspectives, a book to be published, February 2019

Including all the lectures and the workshops of the Re(s)ources conference

Trying to take into account some of the missing resources evidenced by the conference

Particularly from the point of view of teachers' collective work with resources

- Proposing news paths for forthcoming research
- Foreword by Jill Adler
- Afterword by Jeffrey Choppin



4. Perspectives, a symposium for ICMT (Paderborn, 2019, [icmt3.math.upb.de](http://icmt3.math.upb.de))



Third International Conference  
on Mathematics Textbook  
Research and Development

Cross-cultural research on teacher's use of resources

Van Steenbrugge, H., Krzywacki, I, & Hemmi K.

- Comparing naming systems used by Chinese and Ukrainian teachers in describing their resources and documentation work: exploring teachers' resource system (Maryna Rafalskaya, Chongyang Wang, & Luc Trouche)
- Japanese and Swiss pre-service teachers' resources in the cross-cultural collaborative design of a mathematics lesson (Takeshi Miyakawa & Stéphane Clivaz)
- Finnish and Swedish elementary school teachers' interplay with Finnish curriculum resources: an attempt at unraveling tacit cultural practices (Tuula Koljonen)
- A cross-cultural study on teachers' use of digital resources in Sweden, Finland, the U.S., and Flanders (Janine Remillard, Hendrik Van Steenbrugge, Heidi Krzywacki, Kirsti Hemmi, Rowan Machalow, & Tuula Koljonen)
- Supporting teachers' resource use across the cultural and institutional contexts of Mexico, Australia, and South Africa: What have we learned so far? (Jana Visnovska)



## 4. Perspectives, the 25<sup>th</sup> ICMI study (starting February 2019)

### Mathematics teachers working and learning in collaborative groups

Hilda Borko from Stanford University in the USA,  
Despina Potari, from the University of Athens in Greece

The idea of mathematics teachers working and learning through collaboration is gaining increasing attention in educational research and practice.

Across education systems and at all educational levels, mathematics teachers work and learn through various forms of collaboration, which might contribute to their learning and development in different ways.

Efforts to understand what teachers do as they work in collaborative groups, and how this leads to improvement in their practice and expertise, has led to increasing interest in examining the different activities, processes, and contexts for teacher collaboration around the world.

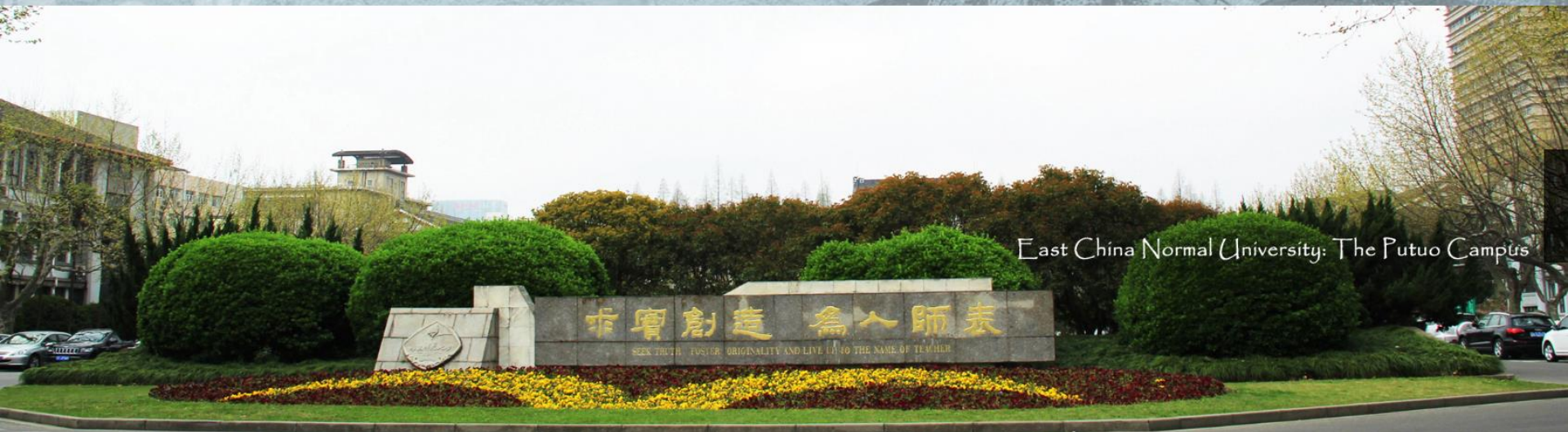


## 4. Perspectives, ICME-14 (ECNU, Shanghai, 2020)



14<sup>th</sup> International Congress on Mathematical Education

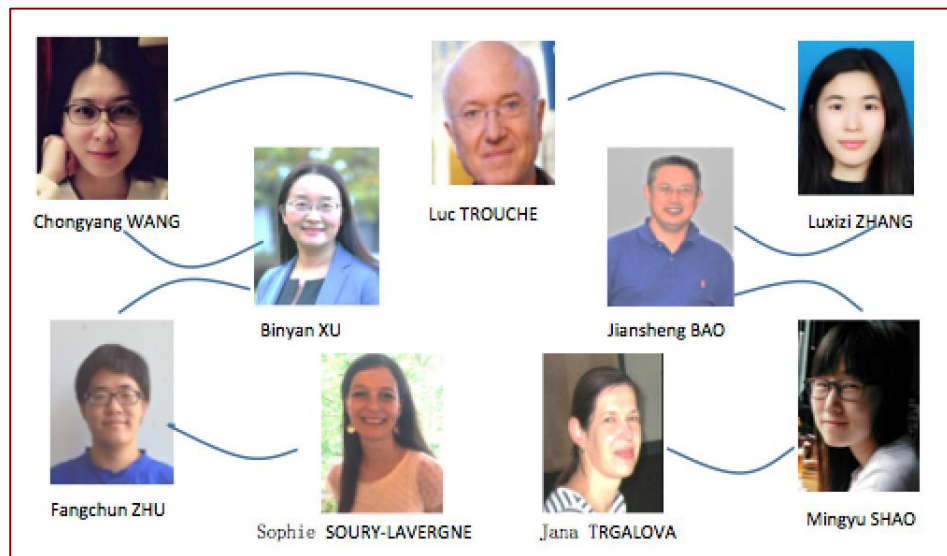
July 12 to 19, 2020 Shanghai, China



- A topic survey group (Rezat-Visnovska), in the thread of the previous TSG, *Research and development on textbooks and resources for learning and teaching mathematics*
- Survey team 3 (Pepin): On teachers' collective work as a regular school practice for teacher development
- Regular lectures: Pr. Fan Lianghuo, Pr. Qi Chunxia, ...

## 4. Perspectives, a French-Chinese seminar in Lyon, next December

Mathematics and education  
Contrasting phenomena  
Building a bridge  
Deepening knowledge



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