Three Basic Dimensions for high quality teaching

Classroom management
both a *condition* for students **getting attentive**
(e.g., through teacher monitoring) and
an *indication* of students **being attentive** (e.g., lack of interruptions)

Supportive Climate
both a *condition* for **students developing learning motivation**
(e.g., teacher being sensitive to individual needs)
and an *indication* of students **being motivated in a self-determined way**
(e.g., low level of achievement pressure).

Cognitive Activation
both a *condition* for **students engaging in knowledge construction**
(e.g., through challenging tasks implemented with adequate pace)
and an *indication* of **students' being engaged in higher order thinking**
(e.g., students providing reasons for their answers).

Praetorius et al. 2018, Lipowsky et al. 2018, Pauli et al. 20

Basic Experiences as goal for learning mathematics

Mathematics lessons should provide three **basic experiences:**

(1) - MODELLING

“Phenomena of the world around us, which concern or should concern u all, from nature, society and culture should be perceived and understoo a specific way.

(2) - MATHEMATICS INSIGHT

Mathematical objects and facts, represented in language, symbols, imag and forms, should be learned and understood as intellectual creations a as a deductively ordered world of their own kind.

(3) - PROBLEM SOLVING

In dealing with tasks to develop problem-solving skills that go beyond mathematics, (heuristic skills)."

→ In every preamble of german curricula and in the national standards

W

Main principles of teaching mathematics

Wagenschein 1977
Winter 1989
Freudenthal 1991
Hiebert & Carpenter 1992

- sense-making by starting with meaningful problem

Wagenschein 1977
Skemp 1976
Hiebert & Carpenter 1992

- developing conceptual understanding

Freudenthal 1973

- students' active engagement

Barzel et al. 2013, Prediger et al. 2

KOSIMA

Kontexte für **Si**nnstiftendes **Ma**thematiklernen

Contexts for Meaningful Learning of Mathematics

Researchproject, 2006 - 2017

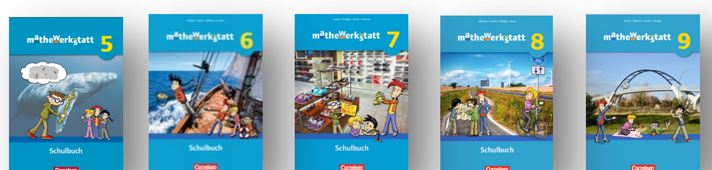
Begin of design research

Different design cycles

2006 2009 2011 2012 2

mathewerkstatt

Self-assessment ℕ,+−x÷



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Barzel et al. 2013, Predig

KOSIMA

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22 Authors in the Design Teams

4 Project leaders and editors

Commercial pu

Corr

13 PhD-students
in the Design Researc

Far more than 100
Master thesis

www.ko-si-ma.de

Approaches & Principles

Underlying approaches for the design:

- Realistic Mathematics Education (Freudenthal 1991)
- Sequentially guided discovery learning with productive failure (Kapur 2010, Loibl et al. 2017)

Design Principles

Develop students' conceptual understanding

(Hiebert & Carpenter 1992; Wagenschein 1977)

Establish high cognitive demands & active epistemic processes

- in mathematization and inquiry processes (Freudenthal 1991, Winter 1989)
- in active knowledge organization processes (Glade & Prediger 2017, Barzel et al. 2013)
- in intelligent exercises (Winter 1984, Wittmann 1994, Foster 2018)

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Realization of design principles

Design principle

Develop students' conceptual understanding

(Hiebert & Carpenter 1992; Wagenschein 1977)

Design element

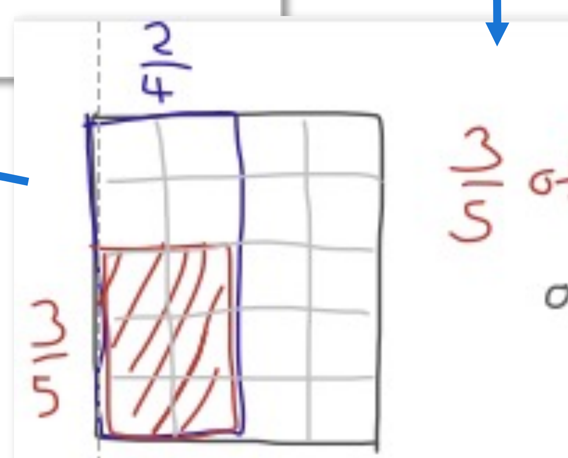
- curricular focus on conceptual understanding throughout all teaching units
- connect multiple representations
- develop learning trajectories in the RME approach to allow students' mear of all mathematical concepts starting from rich imaginable contexts
- start from students' intuitive knowledge

Example for a learning trajectory

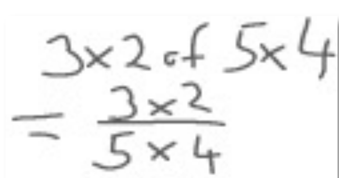
How many children go to school?
In the Chad, only one half of the children go to school and visit grade 1. The rest of the children has to work. Until grade 5, only three fifth of the school children stay at school.

a) Draw a picture for the situation.
b) What is the part of children staying in school until grade 5?

Horizontal mathemati



Vertical mathematization by progressive schematization



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Realization of design principles

Design principle

Use multiple strategies, approaches and representations

(Hiebert & Carpenter 1992; Wagenschein 1977)

Design element

- represent different approaches and representations consistently by four ro
- compare approaches
- connect their approaches

Example How to determine 2/4 of 3/5?

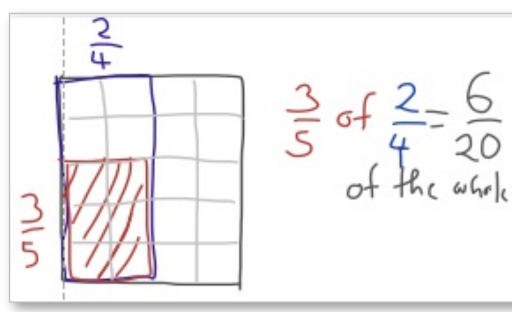
Trial & Error-Till

I first calculate 1/4 of 3/5..

The result times 4 must be 3/5, though 12/20

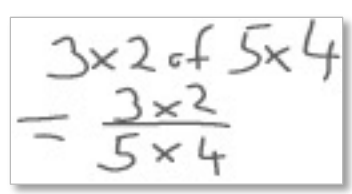
Let us try:

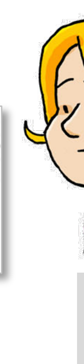
- Try 1/20 → x 4 is 4/20, that is not enough
- Try 5/20 → x 4 is 20/20, that is too much
- Try 3/20, then yes, the result is 12/20



Model Merve

2/4 of 3/5 is one half of 3/5
I imagine a cake with 5 pieces
then I half every fifth,
so I get 3/10





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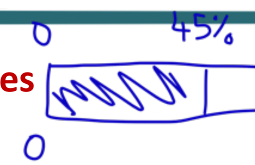
Study-Examples: Local controlled trials for teaching unit

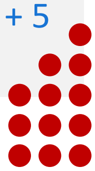


Teaching Unit Elementary Number Theory

- grade 6
- design focus: Strategies for exploration and problem solving

$12 = 3 + 4 + 5$

$12 = 3 \cdot 4$

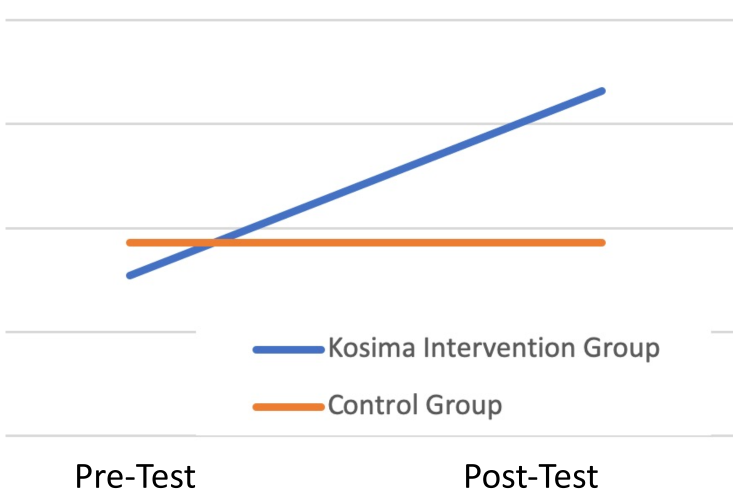


Randomized controlled trial:

- sample: n = 227 sixth graders

Empirical evidence for efficacy:

- significantly different development of problem solving strategies



Philipp K. (2013). Experimentelles Denken Theoretische und empirische Konkretisierung einer mathematischen Kompetenz. Wiesbaden: Springer.

Teaching Unit Percentages

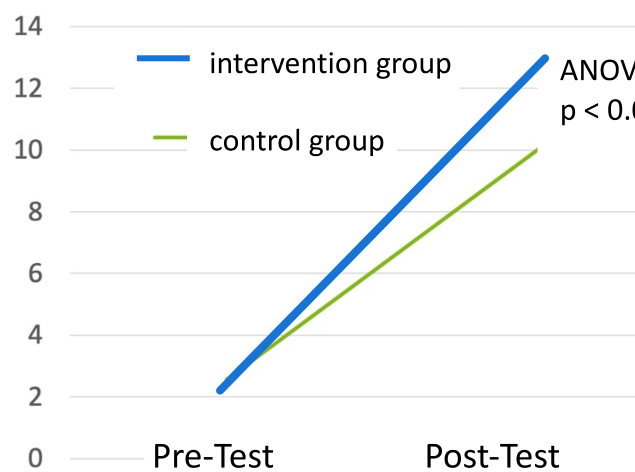
- grade 7
- design focus:
  engaging students in rich discourse pr
  cognitive demands and multiple pathw

Field study:

- sample: n = 655 seventh graders in 79

Empirical evidence for effectiveness:

- significantly different development of conceptual understanding of perce



Prediger, S. & Neugebauer, P. (2021)