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ORIGINAL ARTICLE

Textbook research in mathematics education: development status and directions

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Abstract This paper presents a survey study aiming to systematically examine, analyse and review relevant research focusing on mathematics textbooks and hence identify future directions in this field of research. The literature surveyed is selected from different data sources, including mainly journal articles, research theses and conference proceedings. The survey revealed that important progress has been made over the last few decades in mathematics textbook research, though the major achievement has been concentrated in the areas of textbook analysis (including textbook comparison), and the use of textbooks in teaching and learning. It is overall no longer true that the textbook research in mathematics is "scattered, inconclusive, and often trivial" as described six decades ago; however, the development of research on mathematics textbooks has been unbalanced in different areas. Following the review and discussion, the paper proposes five needed directions for advancing the research in this field.

Keywords Mathematics textbooks · Textbook research · Textbook analysis · Textbook comparison · Textbook use

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1 Introduction

Mathematics textbooks, as supporting materials for teaching and learning of mathematics, have existed since ancient times. It is well known that Euclid's *The Elements* in ancient Greece (about 300 BC) was regarded as "the most successful mathematics textbook ever written" in the West (Merzbach and Boyer 2011, p. 90), while *The Nine Chapters on Mathematics Art* in ancient China (about 200–100 BC) was believed to have "served as a textbook not only in China but also in the neighbouring countries and regions until…1600 AD" (Shen et al. 1999, p. 1).

Compared with the long existence of mathematics textbooks, the study of mathematics textbooks or, more generally, textbook research has a much shorter history. As Cronbach (1955) noted six decades ago, although textbooks were most prevalent in classrooms, research centring on textbooks was "scattered, inconclusive, and often trivial" (p. 4). In the 1980s, the fact that school textbooks remained a largely unexplored field and that more research in this field was needed was increasingly realized by researchers (see, e.g., Freeman and Porter 1989; Graybeal and Stodolsky 1986; Sosniak and Stodolsky 1993). Correspondingly, as Fan (2011) noted, the research on textbooks has grown rapidly over the last three decades.

This paper presents a survey study that aims to systematically examine, analyse and review relevant research, with focus on mathematics textbooks. By doing so, we hope to not only look back at the past development, but also examine the current situation and suggest future directions concerning the research on mathematics textbooks.

This survey article is also in part intended to serve as a background paper for the special issue of *ZDM* with the theme "Textbook Research in Mathematics Education"

though, needless to say, all views expressed in this paper are solely those of the authors.

2 Methods of the survey

We conducted our survey of the literature published over the last six decades mainly through the following three methods.

First, we searched the literature via the so-called world's largest digital library for education literature, the Education Resource and Information Centre (ERIC), primarily obtained using the search terms of "textbook" and "mathematics", and further by several groups of search terms including textbook research, textbook content and textbook analysis, which were summarized as key themes in this area during the reviewing process of the primarily obtained literature.

Second, we systematically examined past issues of research journals in mathematics education to identify the relevant literature, including the following:

Educational Studies in Mathematics
International Journal of Science and Mathematics Education
Journal for Research in Mathematics Education
Research in Mathematics Education
ZDM—The International Journal on Mathematics Education

These journals were selected based on two criteria. Firstly, their scope of publication covers a general range of mathematics education research (not with particular focuses such as on learning or teacher education); and secondly, they were all available to us.

Nevertheless, research articles that were published in other journals also received attention, though the identification and hence inclusion of these articles was mainly through our search of ERIC, not directly from the journals.

Third, attention was also paid to other sources including books, doctoral dissertations and papers presented in conferences mainly through our accumulation of relevant literature (Zhu and Fan 2004) and academic contacts, e.g. via emails. Obviously, due to the wide variety and limited availability of these sources to the research community, our collection of the relevant literature from these sources is by no means complete, which is a limitation of this survey study.

On the other hand, it should be mentioned that most literature identified in the third method, that is, from these other sources, was already collected in the ERIC database and, in some cases, journal articles were based on the doctoral dissertations or conference papers. Therefore, it seems reasonable to argue that our survey of literature is largely comprehensive, though not exhaustive.

As a result of literature searching and screening, it is clear that the main body of the literature we have identified is formed by journal articles and based on original and empirical studies. However, in our survey we did include some non-journal and non-empirical articles, some being more generally about textbook research, based on their relevance.

After we started searching the literature, we soon realized that there had been very little and scattered research published before the 1980s centring on mathematics textbooks, suggesting that the status described by Cronbach (1955) about textbook research was largely unchanged at least till the 1980s, which is apparently related to the fact that textbook research did not receive much attention from the researchers, as mentioned earlier. Hence below we mainly focus on the relevant literature since the 1980s.

Table 1 displays the sources of literature surveyed in this study, which clearly shows the rapid growth of research interest and outcomes in this area over the last three decades.

We must point out, although we tried to make the survey as comprehensive as possible, it remains possible that some important research work in this area has been missed in the selection process. This is due to a variety of reasons including the scope and focus of the study and the fact that not all research is accessible via ERIC or published in journals or conference proceedings, which is always a challenge for such a survey study. In addition, our survey is mainly limited to research literature in English, though some attention was also paid to non-English literature (e.g. in Chinese); however, it was not our focus.

After we selected the literature, we classified all the articles into four categories using the following framework established for this study (see Table 2). The framework is primarily based on the focus of these articles identified.

An initial screening of all the empirical studies identified reveals that most of them can be classified into the second or third category. However, we also found a fourth category, "other areas", necessary in order to broadly

Table 1 Sources of literature surveyed in the study

	Before 1980	1980–89	1990–99	2000–09	2010–2012	Total
Journal articles	2	18	21	26	9	76
Other publications	4	4	8	11	8	35
Total	6	22	29	37	17	111

Table 2 A framework for classifying the literature on textbook research in mathematics

	Description
Category 1 **Role of textbooks**	Literature about role of textbooks in mathematics teaching and learning. This category is necessary to reflect the focus and debate of most philosophical or non-empirical articles centring on the role of mathematics textbooks
Category 2 **Textbook analysis and comparison**	Studies focusing on analysing the concerned features of mathematics textbooks under study and, in the case of textbook comparison, comparing the similarities and differences of two or more series of mathematics textbooks
Category 3 **Textbook use**	Studies focusing on how textbooks are used by teachers and/or students; in other words, how textbooks shape the way of teaching and learning of mathematics
Category 4 **Other areas**	Broadly including all the other studies such as those about electronic textbooks and about the relationship between textbooks and students' achievement

include all the other studies such as those about electronic textbooks, which are fairly new and worth a special consideration, and about the relationship between textbooks and students' achievement. Nevertheless, the number of such studies is very few, implying that a further classification is neither really feasible nor necessary.

Figure 1 displays the distribution of all the individual studies in the last three different areas based on the framework as explained above. For convenience, textbook comparison is listed separately from textbook analysis in the diagram, though textbook comparison can be considered as a subset of textbook analysis (see more in Sect. 4 below). In addition, the figure does not include some non-empirical literature and the category about the role of textbooks (hence n = 100 instead of 111), as the relevant discussions or debates are often not presented as individual research studies; instead, they are provided as discussion papers or as part of and hence serve the main studies.

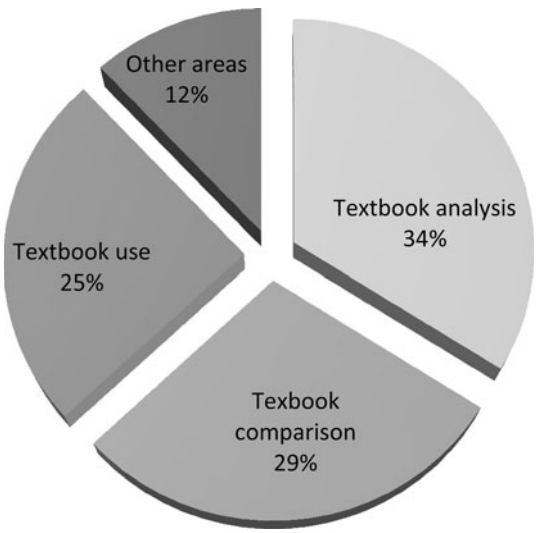

Fig. 1 Distribution of empirical studies on mathematics textbooks surveyed in different focus areas (n = 100)

A reliability check by comparing the results obtained by the first and third researchers of this study from their independent coding/classifying work on the first 50 empirical studies shows a 96 % inter-rater consistency, therefore we believe the coding result is highly reliable, which also indicates that the coding is fairly straightforward.

Figure 1 shows that the examined studies in textbook research have been mainly concentrated in the area of textbook analysis, which occupies 63 % of all the studies identified (including textbook comparison accounting for 29 %), and textbook use (25 %), while much fewer are found in the other areas (12 %).

Below we shall start with the literature concerning the role of textbooks in teaching and learning of mathematics, followed by textbook analysis and comparison, textbook use, and other areas. Obviously, due to the large body of research literature published in different areas, the research work we can report below must be limited and selective. The paper ends with a summary and discussion about future directions for textbook research in mathematics education.

3 Role of textbooks in teaching and learning

The survey revealed that the important role of textbooks in teaching and learning has been widely recognized by researchers.

Researchers have generally agreed that textbooks as a major conveyor of the curriculum play a dominant role in modern education scenes across different school subjects. Moreover, in mathematics, Robitaille and Travers (1992) argued that a great dependence upon textbooks is "perhaps more characteristic of the teaching of mathematics than of any other subject" (p. 706).

In relation to such importance, Sosniak and Perlman (1990) pointed out that the power of textbooks lies in their ability to serve as resources which introduce readers to worlds which are not immediately obvious or cannot be experienced directly. In particular, textbooks have their power in providing an "organized sequence of ideas and information" to structured teaching and learning, which guide readers' "understanding, thinking, and feeling" as well as "access to knowledge which is personally enriching and politically empowering" (p. 440). Apple (1986) also

suggested that "it is the textbook which establishes so much of the material conditions for teaching and learning in classrooms … and … often defines what is elite and legitimate culture to pass on" (p. 81).

Based on an empirical study about teaching of mathematics using different textbooks, Fan and Kaeley (2000) found that teachers using different types of textbooks displayed different styles of teaching strategies, and concluded that textbooks appear to play a role in teachers' pedagogy by conveying pedagogical messages and providing an encouraging or discouraging curricular environment for them to employ different teaching strategies.

Researchers' conceptualization about the relationship between the textbooks and curriculum is particularly noteworthy. In his remarkable book on comparison of textbooks for the Third International Mathematics and Science Study (TIMSS), Howson (1995) pointed out that textbooks were one step nearer classroom reality than a national curriculum. Schmidt et al. (1997) further argued that "two worlds—that of official intentions and that of actual classroom activities—are tied together, in part, by textbooks" (p. 53).

Regarding the role of textbooks in the curriculum, a conceptualization (Fig. 2) proposed by the TIMSS group of researchers deserves special attention. Although textbooks have been commonly treated by educational researchers as part of the intended curriculum, the TIMSS group defined textbooks as a fourth level, the potentially implemented curriculum (Schmidt et al. 2001; Valverde et al. 2002), to add to its original tripartite concept model of curriculum, that is, intended curriculum, implemented curriculum, and attained curriculum, which is well known to educational researchers.

About the role of textbooks in different levels of curriculum, Valverde et al. (2002) further explained that "[t]extbooks are designed to translate the abstractions of curriculum policy into operations that teachers and students can carry out. They are intended as mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms" (p. 2).

More recently, from a research perspective, Fan (2011) proposed a conceptual framework which conceptualizes textbooks as an intermediate variable in the context of education and accordingly he defined mathematics textbook research as disciplined inquiry into issues about mathematics textbooks and the relationships between mathematics textbooks and other factors in mathematics education.

4 Textbook analysis and comparison

Textbook analysis is a broad term primarily including (1) analysis of a single textbook or a series of textbooks, which often focuses on how a topic or topics are treated or how a particular idea or aspect of interest is reflected in the textbooks, and (2) analysis of different series of textbooks from the same country or, more frequently, different countries, often with focus on identifying their similarities

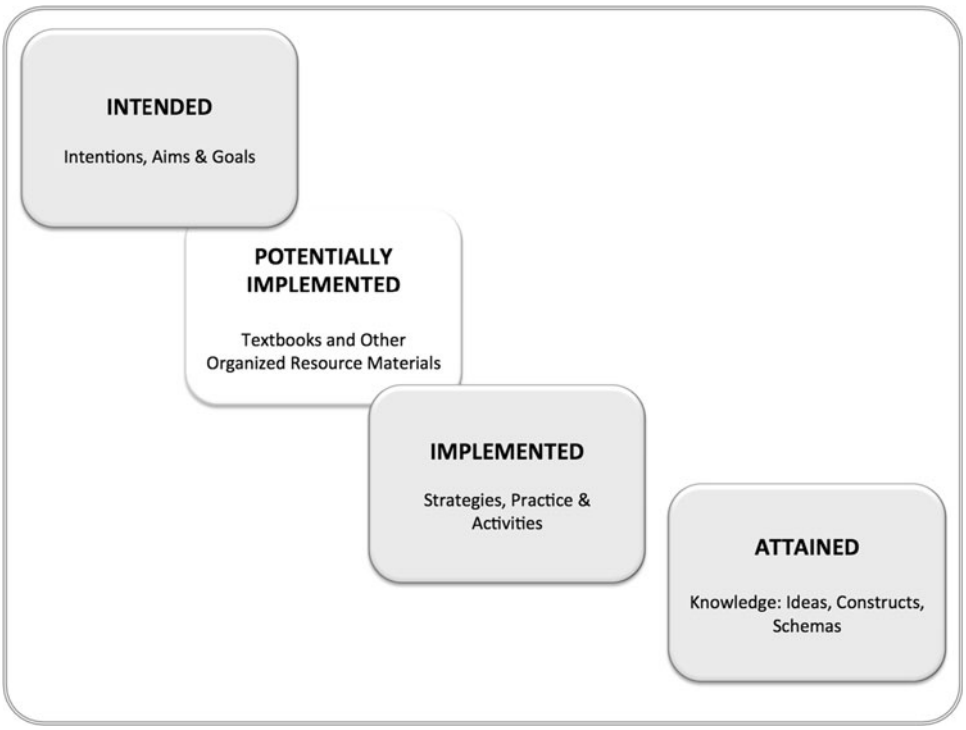

Fig. 2 A conceptualization proposed by the TIMSS group of researchers (Valverde et al. 2002)

and differences. The latter is also termed textbook comparison. Obviously, textbook comparison must be based on textbook analysis of each individual series of textbooks; hence it can also be termed textbook comparative analysis, a main reason for us to create this category.

Johnsen (1993) once generally defined textbook analysis as product-oriented approach. He surveyed the literature on textbooks from Germany, France, UK, USA and, most notably, Scandinavian countries. It revealed that textbook research using a product-oriented approach, which focused on subject matter knowledge, had dominated the field, while little research had been done on writing, development and distribution of textbooks (i.e. a process-oriented approach), and research about textbook use (i.e. a use-oriented approach) had started to receive slightly more attention.

Consistently, our survey also revealed that, as shown in Fig. 1, among the existing research, the largest body of work (63 %) is found on textbook analysis and comparison.

Considering the scope and length of this paper, below we shall briefly present an overall picture of the research, subdivided into five aspects, using a selected set of studies. The five aspects of textbook analysis are: (1) mathematics content and topics; (2) cognition and pedagogy; (3) gender, ethnicity, equity, culture and value; (4) comparison of different textbooks; and (5) conceptualization and methodological matters. It should be noted that these aspects are often intertwined within the same study, for example a comparison study can be about mathematics content or topics in different textbooks. In these cases, discretion is used by the researchers based on its focus and the need of our discussion.

4.1 Mathematics content and topics

Most studies in textbook analysis, as revealed in the survey, have focused on issues about how different mathematics content or topics have been treated in the textbooks concerned. As noted by researchers (e.g. Johnsen 1993), this result is not surprising.

An early study which is often referred to was done by Flanders (1987), examining three widely used series of US mathematics textbooks for grades K-9 to analyse how much of the content was new rather than review of old content. The study revealed that new content introduced in each grade tends to decrease from Grades 3 to 8 with the lowest amount found in Grades 6, 7 and 8. In particular, in two of the three series, only less than 40 % of the content was new in these three grades. The study raised a serious problem for textbook developers and teachers: would students be less motivated if they found that so much content was old and expected that much of the new content to be learned would be repeated somewhere in the future? Interestingly, similar repetition was also found in a recent study by Pickle (2012) about the treatment of statistical content in four series of US mathematics textbooks for middle school students.

Reys et al. (1996) studied how three Japanese primary textbook series presented and developed computation. They found that the goals were similar but the methods and materials of presentation were very different and, moreover, a much higher level of computational facility at the end of grade three was expected of Japanese students than of US students.

Levin (1998) examined how fractions and division were presented in American elementary, middle school, and algebra textbooks. She found that most textbooks examined showed no connection between fractions and division, and fractions were rarely defined or discussed as division. The study also revealed considerable differences in the presentation and placement of these topics in textbooks, suggesting researchers' lack of agreement about how to teach these two topics. Levin argued that because connections between fractions and division are largely absent in textbooks, teachers must fill in the gaps.

Focusing on proportional reasoning and drawing on relevant literature as well as mathematics curriculum reform principles underpinning the mathematics syllabus in Queensland, Australia, Dole and Shield (2008) established a framework of criteria for textbook analysis, and used it to analyse two eighth grade mathematics textbooks and examine how knowledge connections and proportional reasoning were promoted. The analysis revealed a predominance of calculation procedures adopted in the textbooks, with relatively few tasks and explanations to support conceptual understanding.

Stylianides (2009) analysed the opportunities in a series of US mathematics textbooks for students to be engaged in reasoning-and-proving, and found that among all 4,855 tasks, only about 40 % of them were offered with at least one such opportunity for students, and more than 50 % with no opportunity at all for student engagement into such academic activities.

Similarly, Stacey and Vincent (2009) examined the reasoning presented in nine Australian eighth grade textbooks, and found that most of them provided explanations for most of the seven topics analysed, though these explanations were mainly used for rule derivation or justification rather than as thinking tools. In addition, the differences about the legitimacies of deductive and other modes of reasoning were generally not made.

4.2 Cognition and pedagogy

Concerning how cognitive skills were demanded in the US elementary and secondary mathematics textbooks, Nicely

(1985) used an original 10-level classification of thinking activities and revealed a general decline both in the amount of material demanding student involvement and in the percentage of that material requiring higher-order thinking. The study concluded that the mathematics textbooks are not enough to actively involve students in the development, practice and acquisition of higher-order thinking skills. Similarly, the inadequacy of provision for activities demanding higher-order thinking was also reported in another study by Nicely et al. (1986) with focus on decimal problems in 16 elementary textbooks.

Breakell (2001) analysed 176 mathematics textbooks and reference books used in primary and secondary schools in England and Wales in the late 1950s and early 1960s, and found that "a powerful diet" of traditional content material was presented in a traditional manner in 30 out of 51 textbooks or textbook series. The researcher argued that the results from the data reflect some degree of accuracy in terms of both the content and the methods of teaching favoured by the teachers of the day.

Vincent and Stacey (2008) investigated the so-called 'shallow teaching syndrome' by examining the problems on three topics in nine eighth grade textbooks in Australia. The analysis focused on procedural complexity, type of solving processes, degree of repetition, proportion of 'application' problems and proportion of problems requiring deductive reasoning. It was found that there was broad similarity between the characteristics of problems in the textbooks and these shown in the 1999 TIMSS Video Study lessons of Australian eighth grade mathematics lessons, which employed a high proportion of problems of low procedural complexity, with considerable repetition and an absence of deductive reasoning. Nevertheless, considerable differences were also observed between textbooks and between topics within textbooks. In some textbooks, including some best-selling ones, the balance was too far towards repetitive problems of low procedural complexity.

Problems and problem-solving appeared to be the most popular non-mathematics-specific topics in textbook analysis (e.g. Fan 1998; Fan and Zhu 2000; Li 2000; Nibbelink et al. 1987; Sun 2011), given that problem solving has been the central theme of mathematics education since the early 1980s, from both cognitive and pedagogical perspectives. Many studies in this sub-area involved comparison of textbooks.

Mayer et al. (1995) investigated how mathematics textbooks in Japan and the USA teach mathematical problem-solving. By analysing lessons on addition and subtraction of whole numbers in three seventh grade Japanese textbooks and four US textbooks, they found that the Japanese textbooks devoted 81 % of their space to explaining the solution procedure for worked examples, compared with 36 % in the US textbooks. On the other hand, the US textbooks devoted more space to unsolved exercises (45 %) and irrelevant illustrations (19 %) while the Japanese textbooks only devoted 19 and 0 % of the space, respectively. The results were found consistent with classroom observations showing that Japanese classrooms tend to emphasize the process of problem-solving more effectively.

Zhu and Fan (2006) compared how selected textbooks from China and the United States at Grades 7 and 8 represent various types of mathematics problems such as routine problems vs. non-routine problems, open-ended problems vs. close-ended problems, and traditional problems vs. non-traditional problems. It revealed that in both countries, more than 96 % of problems were routine and traditional, more than 93 % were close-ended problems, and more than 92 % were irrelevant to real-world situations. The results also showed that the problems in the Chinese textbooks were overall more challenging in terms of the steps involved in the problem solutions, while the US textbooks provided considerably more application problems, especially authentic ones, and problems with visual information.

Furthermore, Fan and Zhu (2007) compared how school mathematics textbooks in China, Singapore and USA represent problem-solving procedures in two layers: general strategies, which adopt Pólya's four-stage problem-solving model, and specific strategies consisting of 17 different problem-solving heuristics, for example, 'acting it out', 'looking for a pattern' and 'working backwards'. The results showed that the textbooks all demonstrated some general problem-solving procedures and heuristics, but this aspect was much more explicit in the Chinese textbooks. The study also found that there existed considerable gaps between national syllabuses/curriculum standards and the textbooks. Clearly, to understand why there are such gaps and how to close the gaps is a challenge for researchers, textbook developers, policy makers and teachers.

4.3 Gender, ethnicity, equity, culture and value

Some earlier studies were related to gender and equity issues. Prompted by an awareness and people's general concern about the sexism issue in curriculum and textbooks, Kuhnke (1977) investigated two series of US elementary mathematics textbooks, and concluded that positive efforts had been made to eliminate sex role stereotyping. Similar positive conclusions were also obtained by Nibbelink et al. (1986) who analysed problems presented in the third to sixth grade US mathematics textbooks and found that the textbooks were non-sexist, though not anti-sexist.

Garcia et al.'s (1990) study was more broadly concerned with whether elementary mathematics textbooks enhanced

a socially active mathematics education programme, and revealed that young females as well as minorities were adequately portrayed in five US mathematics textbook series examined, though they found that the representation of careers in the textbooks was less than adequate.

In the Australian context, Clarkson (1993) analysed 18 mathematics textbooks with focus on gender, ethnicity and equity. The study found that most segments of the books examined did not refer to people, 45 % of people depicted were male while 39 % were female, and most people depicted were shown as active not passive. Moreover, only 8 % of instances showed a person who was non-Anglo-Australian.

Today, few textbook analyses focus on the aspect of gender, ethnicity and equity issues, and this seems in part due to the progress made in these issues. Arguably, however, these issues are still worth reasonable attention, especially for textbook developers and policy makers.

Related to issues of gender, ethnicity and equity, a broader issue is about culture and values in mathematics education. Seah and Bishop (2000) investigated mathematical and mathematics educational values conveyed in the seventh and eighth grade mathematics textbooks in Singapore and Victoria, Australia. Viewing mathematics textbooks as source and medium of values portrayal, they found that the text in both countries conveyed "the mathematics educational values of formalistic view, instrumental understanding, theoretical knowledge, specialism and evaluating, with greater emphases than their respective complementary values". Moreover, they noted that although Singapore and Victoria are both multicultural societies, it is only in the Victoria textbooks that students are likely to be invited to contribute their own cultural experiences and knowledge (see also Seah 1999).

Undoubtedly, the issue about culture and values in mathematics textbooks has importance for textbook evaluation, use and development. The challenge remaining is to find and reach consensus about what should be the criteria concerning the correct or balanced representation of desirable culture and values, and based on what grounds (see also Ka and Leung 2012; McBride 1994).

4.4 Comparison of different textbooks

Here we focus on international comparisons. So far, the largest-scale comparative analysis was done as part of the TIMSS study in the 1990s and it is worth particular attention. It was the first time that textbooks were perceived as an independent comparative variable in such large-scale international comparative study series.

The TIMSS researchers compared mathematics (and science) textbooks used in nearly 40 participating countries with focus on five broad areas: (a) the nature of the pedagogical situation posed by the textbook; (b) the nature of the subject matter such as the number of topics, the degree of abstraction, and the complexity of the topics; (c) sequencing of topics; (d) the physical characteristics of the textbooks, for example, size and length; and (e) the complexity of student behaviour the textbook is intended to elicit (Valverde et al. 2002, p. 14). Twelve variables were chosen to represent these five areas. For example, the measures of the percentage of each examined textbook devoted to providing exercises, mathematics activities, worked examples, etc. were chosen to represent the first area described above. Based on rather detailed analysis of hundreds of textbooks with many specific results, the researchers' overall conclusion was that the textbooks from different education systems varied in many ways and "they exhibit substantial differences in presenting and structuring pedagogical situations and these differences are systematically related to country, grade level and subject matter differences" (Valverde et al. 2002, p. 17).

Other textbook comparison studies, some of which were mentioned above, were usually conducted at a small scale and by individual researchers, including research students through their doctoral dissertations (also see Grevholm 2011).

An early textbook comparison study was done by Stigler et al. (1986), focusing on addition and subtraction word problems in four US series and one former Soviet series of elementary textbooks. It was found that all the US textbooks were similar to each other, but the Soviet textbook was different from the US ones in several ways. In particular, Soviet textbooks had much more variety of problem types, while the US ones focused more on the problems easiest to solve. Moreover, Soviet textbooks presented different problems throughout the text rather than focusing more difficult problems in a specific section of the text.

A related study by Fuson et al. (1988) looked at the grade placement of addition and subtraction topics in Japan, mainland China, the Soviet Union, Taiwan and the USA. It revealed that there was remarkable uniformity in grade placement in the textbooks from Japan, mainland China, the Soviet Union and Taiwan, but the US textbooks differed substantially. Also focusing on addition and subtraction, Carter et al. (1997) compared how school textbooks (Year 6 and 7) in China and the USA presented integer addition and subtraction, and found substantial differences between them.

Harries and Sutherland (1999) compared mathematics textbooks from England, France, Hungary, Singapore and the USA about how the books introduce multiplication and division at the lower primary level. In highlighting the strength of the Singaporean, Hungarian and French texts in using appropriate representations, they suggested the

necessity for England to pay more attention to the use of all forms of representations.

Focusing on mathematics textbooks in English, French and German, Pepin and Haggarty (2001) analysed one of best-selling mathematics textbook series from each of these countries. They found that the structures of mathematics textbooks in these three countries were quite different. While the French textbooks contained activities, essential exercises, and accommodating exercises, aiming to guide students to new notions, German textbooks were constructed by introductory exercises and the main notion followed by an extensive range of exercises. Comparatively, British books seemed simple, with straightforward questions put forward before the worked examples. According to the researchers, the differences are related to different educational contexts and traditions. In particular, the unique part found in the structure of French textbooks, namely, activities (small investigations), seems to "fit in with Piaget's notions of constructivism and their associated teaching approaches".

4.5 Conceptualization and methodological matters

Along with the growth of studies in textbook analysis, conceptualization and methodological issues have received increasing attention from researchers. Below we briefly present four of them from different angles.

Pepin and Haggarty (2001), based on their review of the relevant literature, presented a framework which classified the analysis of textbooks into four main areas: "mathematical intentions of textbooks", "pedagogical intentions of textbooks", "sociological contexts of textbooks", and "cultural traditions represented in textbooks".

Bao (2002) proposed a composite difficulty model in his doctoral study to measure and compare the difficulty levels between the UK and China mathematics curriculum, which included textbooks as intended curriculum. The model consists of five factors, namely, topic coverage, investigation, context, reasoning and computation. The study focused on the eighth grade and found that the Chinese curriculum is overall more difficult than the UK curriculum, except in the factor of "context".

Kim (2009) conducted interviews with 21 teachers (South Korea, 11; US, 10), as well as with seven textbook authors (South Korea, 4; US, 3) to develop a framework for the evaluation of non-textual elements, such as photos and graphs, in mathematics textbooks. Validated through measuring four Korean and three US mathematics textbooks, the framework comprises five elements: accuracy, connectivity, contextuality, simplicity and aesthetics.

Charalambous et al. (2010) classified textbook analysis into three broad categories as horizontal, vertical and contextual. In the horizontal analysis, the textbook is examined as a whole with a focus on general textbook characteristics (e.g. physical appearance and the organization of content). However, this approach has been criticized for overlooking fundamental differences in the learning opportunities. The vertical analysis is about the treatment of a single mathematical concept or topic in the textbook and it has been argued that this approach could potentially overlook the relationship between the examined topic and others in the same book. The contextual analysis is about textbook use, which falls in the domain of the implemented curriculum. In our view, the research about textbook use should not be regarded as part of textbook analysis, as the latter is essentially document analysis.

To conclude this section, it appears clear that most studies in textbook analysis have consistently revealed, to a greater or lesser degree, the inadequacy of textbooks in presenting mathematics content, topics and problem-solving; remarkable differences were found in textbooks from different series and particularly from different countries, which seems to us to point not only to the lack of consensus in textbook development, but also to the inseparability of textbooks from the cultural and social background. Moreover, the available textbook analysis studies have also revealed the gaps between textbooks and intended curriculum. These results indicate both challenges and need for researchers, curriculum developers, policy makers and school teachers to conduct further research and action.

Finally, it can be noted that most textbook analysis studies were conducted at the primary level, as revealed earlier (see also Alajmi 2012; Bierhoof 1996; Newton and Newton 2007; Stigler et al. 1982). Considerably fewer studies were found at the secondary level (see also Jones and Tarr 2007; Samimy and Liu 1997), and even fewer were at both primary and secondary levels, which also implies the challenge and need for further research in this area.

5 Textbook use in teaching and learning

Regarding the traditional methods of investigating textbooks, Gilbert (1989) criticized the reliance of research on text analysis with little attention to the context of textbook use. He commented that text analysis might be able to predict, but can never conclude with confidence, the actual classroom use of texts. Hence, it is important to examine textbooks not only in terms of their content and structure, but also their use in real classrooms. Many other researchers have also advocated the investigations of textbooks as they are used in the classroom (e.g. McCutcheon 1982; Stodolsky 1989).

A notable earlier study was done by Krammer (1985), who conceptualized the textbook as a classroom context

variable and compared the teaching of Netherlands teachers using three different mathematics textbooks. Based on the data collected through classroom observation, tests and questionnaires for students and teachers from 50 eighth grade classes in 17 college-bound schools, the study revealed a significant difference in teaching practices among the three textbook user groups in the frequency of posing higher-order questions, the amount of seatwork, the amount of academic conversation, and the students' perception of remedial help; consistency was found between the teaching practices and the textbook features. Interestingly, Krammer raised the question whether such consistency is because the teachers followed the textbooks or because the teachers chose textbooks that resembled their preferred teaching style (also see Zahorik 1991). It is worth noting that, even today, this question seems still relevant and largely unanswered.

To investigate teachers' use of primary textbooks and the overlap between the textbook content and the teaching content, Freeman and Porter (1989) analysed four mathematics teachers' daily logs and quantitatively measured the content overlap between the same textbook they used and lessons they delivered throughout an entire school year. Striking differences were found in the content overlap, their time allocation, grouping practices and achievement standards among the four teachers. The result revealed the crucial role that teachers played in the use of textbooks in classrooms and argued that the differences in teachers' implementation of textbooks could considerably explain the differences of the effectiveness of teaching and the outcome of students' learning.

Remillard (1999) studied two elementary teachers' use of the textbook and established a three-arena model of teachers' curricular construction based on the findings. Echoing the noticeable disparities between the textbook and its classroom implementation as revealed by Freeman and Porter (1989), she called for attention to teachers' engagement in the curricular construction since the actual curricular content exposed to students was dramatically decided by teachers' personal beliefs and their decisions made in the three arenas: design (selecting and designing mathematics tasks), construction (enacting selected/designed tasks and responding to students) and curriculum mapping (determining the organization and content of curriculum).

Zhu and Fan (2002) investigated how Singapore mathematics teachers used two most commonly used textbooks at the lower secondary level. Using the data collected through a questionnaire survey from 28 teachers in eight schools, a stratified random sample from all the 110 secondary schools using the textbooks, they found that textbooks were overall an important but not the only resource for teaching, and there were largely no significant differences between teachers with different genders, experiences, and from different schools in their use of mathematics textbooks.

A similar study was conducted by Fan et al. (2004) in China on textbook use by teachers, as well as students, within and beyond classrooms. Using a questionnaire survey, classroom observation, and interview on a sample of 36 mathematics teachers and 272 students from 12 secondary schools, the study found that teachers treated textbooks as a main but not the only source for their teaching, while students had greater dependence on textbooks as the major learning source both within and beyond the classroom. Moreover, students differed significantly in their way of using textbooks, while there was no significant difference in teachers' use of textbooks across different genders, experiences, schools and regions.

Nicol and Crespo's study (2006) was about pre-service teachers' use of curricular materials. The study involved four prospective mathematics teachers enrolled in a Canadian teacher education program and collected data with course-work content analyses, semi-structured interviews, and classroom observations during their practicum. It revealed that these pre-service teachers' understandings of textbook use were challenged and changed during and after their classroom practice. Interestingly, similar observations were also reported in an earlier study by Ball and Feiman-Nemser (1988).

Concerning students' reading comprehension of different types of mathematical texts, Österholm (2006) recruited 61 secondary students and 34 university students to test their understanding of a mathematical text with or without symbols and a history text. By comparing students' prior knowledge with their reading test results vertically and comparing their comprehension of the three types of texts horizontally, the study showed a similarity between student understandings of the mathematical text without symbols and the historical text and a difference in understanding between the two types of mathematical texts. He argued that the training of reading skills in understanding general texts and that in mathematical texts with symbols were both important and the latter should be treated differently from the former (also see Dowling, 1996; Österholm 2008; Weinberg and Wiesner 2011).

Using the concept of "implementation fidelity", McNaught et al. (2010) conducted a three-year project on teachers' use of two types of mathematics textbooks from the perspectives of both teachers and researchers in the US context. The result found that teachers tended to assign fewer problems to students than the textbook authors recommended and covered less than 70 % of textbook contents on average. Similarly, Eisenmann and Even (2011) conducted a comparison between the ways in which one teacher implemented the same algebra content of one

textbook in two seventh grade classes taught by her and found notable differences between her ways of textbook implementation in the two classes.

Randahl (2012) explored the role of mathematics textbooks at the tertiary level, which has received very limited exposure in previous research. The study collected data from first-year engineering students, who were taking a basic calculus course, through questionnaires, observations and interviews. The results showed that the textbook was used to a very low degree and mainly perceived as a source of tasks. The researcher argued there is a need for greater awareness about the use of mathematical textbooks in meaningful ways at the tertiary level.

It should be noted that a number of mathematics educators have also paid particular attention to conceptual and methodological matters about the research into textbook use. Pepin and Haggarty (2001) summarized six main researchable themes: (1) use or not use of textbooks; (2) authority of the textbooks; (3) the users of the textbooks and the decision makers about the users; (4) the ways of using textbooks and the decision makers about the usage; (5) teachers as mediators of the texts; and (6) the influence of national culture on classroom practices. Remillard (2005) emphasized that how teachers interact with curricular resources is critical to understanding their use of curriculum resources, for which she proposed a framework centring teachers' interactions with curriculum materials (also see Remillard 1999). In addition, Rezat (2006) proposed a model of textbook use from the perspective of activity theory, which emphasizes the use of a textbook as an activity situated in classroom settings being both object-oriented and collective.

Using a model of teachers' textbook use consisting of five levels (misuse, mechanical use, routine use, refinement use and creative use) in four aspects (comprehending and studying textbooks, integrating textbooks, applying textbooks and making judgement on textbooks), Kong and Shi (2009) measured the level of five primary teachers' textbook use in China before and after an intervention in which the researchers provided teachers with professional guidance on how to prepare, implement and reflect on their use of textbooks. From the result, they claimed that the level of textbook use model is valid and accurate, and could help improve teachers' textbook-use skill and professional development.

Love and Pimm (1996) once argued that there was "a dearth of research" about textbook use, mainly because of the difficulty in collecting data from classrooms for such studies. The review presented above shows that this situation has considerably changed since then, and researchers have paid increasing attention to the use of mathematics textbooks.

On the other hand, it is clear that most of the studies were carried out in a small scale by individual researchers, were exploratory in nature, and were on textbook use by teachers, with much less on textbook use by students. In addition, despite the progress made in conceptual and methodological matters about research on textbook use, overall it appears to be still at an initial stage and is far from adequate (see more in Fan 2011). Further research on a larger scale, and confirmatory research with experimental design, and on students' use of textbooks, is much needed.

6 Textbook research in other areas

While the bulk of textbook research has been on textbook analysis and comparison, and the use of textbooks in mathematics education, researchers have also paid attention to some other areas centring on textbooks, though, as Fig. 1 shows, the number of such studies is considerably fewer.

Some researchers have also looked into the relationship between textbooks and tests or students' performance in mathematics. For example, Flanders (1994) studied relationships between intended, implemented, and tested curricula of 84 American eighth grade mathematics classes based on data from the Second International Mathematics Study. It showed that the test was not representative of the curriculum defined by textbooks.

Similar results were also reported in an earlier study by Freeman et al. (1983), who investigated the influence of five different styles of textbook use on the congruity of textbook and standardized test. Furthermore, Chandler and Brosnan (1995) compared contents covered in seven mathematics textbook series used from Grade 1 to 8 and a statewide ninth grade mathematics proficiency test in USA, and found that the areas of greatest mismatch were arithmetic, measurement and algebra.

Concerning learning opportunities offered in mathematics textbooks and their effects on the learning outcomes, Xin (2007) tested 57 US and 54 Chinese students with learning difficulties and correlated their performance with the distributions of various types of word problems in the textbooks used by these students. The results revealed that the Chinese students obtained a high score, while the comparative analyses of textbooks showed an unbalanced distribution of word problems throughout the US textbook. Such imbalance was regarded as an essential factor contributing to the US students' performance. Xin called, rightly, for more follow-up qualitative evidence, such as classroom observation, to triangulate the findings. Nevertheless, as we can see, this type of study is again more exploratory rather than confirmative since the textbooks are only one of the many factors affecting students' learning outcomes.

Regarding mathematics teachers' preferences about textbooks, Shield (1989) conducted a survey of 28 teachers in three Australian secondary schools about their preferences in textbook characteristics as well as in the use of textbooks by them and their students. The study found that teachers valued more the characteristics that were related to the students' use of the textbooks, most importantly for students' exercises both in class and for homework. In other words, mathematics teachers valued the practicality of the textbooks for teaching and learning.

Focusing on the relationship between students' perceptions of mathematics curriculum and their sense of identity in mathematics learning, Macintyre and Hamilton (2010) did a study on two series of Scottish textbooks with 48 students aged 14–15 evenly distributed in four groups. It revealed that the learners' engagement and success in mathematics were influenced by content selection and presentation of textbooks. The study indicated the potential impact of textbook design on textbook users and their perceptions of the subject matter.

When we started searching for literature on textbook research in mathematics education, we noted there was a call for research on textbooks in ICME10 Discussion Group 14 (Fan et al. 2004) which explicitly included "electronic textbooks" and we expected to identify some research on electronic mathematics textbooks. However, the result is disappointing with research with clear research questions, methods and results being virtually non-existent.

Nevertheless, we did note some initial meaningful research-related work in this connection. For example, Sinclair (2003) shared experiences and discussed issues about interactive mathematics textbooks, while Shepherd and van de Sande (2011) described how 30 university students read from an online mathematics textbook in a precalculus course. Gould (2011) employed examples from both printed textbooks and available electronic ones and discussed issues about how educational design features can possibly help align the medium of presentation with the content of electronic mathematics textbooks, emphasizing that digital texts can provide different affordances and constraints to learning mathematics.

Overall, it is apparent that research centring on electronic textbooks in mathematics education is still at a preliminary stage, despite the rapid growth of the availability of electronic mathematics textbooks for classroom use.

7 Summary and future directions

From the review and discussion presented above, we can see clearly that the status of textbook research in mathematics education has considerably changed compared with the status of six decades ago. With a rapidly increasing attention from mathematics educators, and the growth of the number of studies particularly over the last three decades, as shown in Table 1, it is overall no longer true to claim that the research centring on textbooks in mathematics is "scattered, inconclusive, and often trivial" as Cronbach judged with reference to general textbook research (Cronbach 1955, p. 4).

On the other hand, we can also see that the development of research on mathematics textbooks has been unbalanced in different areas. In general, researchers have developed better understanding of the role of textbooks in mathematics curriculum, teaching and learning. Many and major research studies have been conducted in the area of textbook analysis, textbook comparison and the use of textbooks, the last of these being mostly use by teachers but some also by students. A small number of researchers have also made efforts to look into some other areas, though research in these areas is still indeed scattered, inconclusive and often trivial.

To further advance the research on mathematics textbooks, we think that, firstly, it is necessary for researchers to establish a more solid fundamental conceptualization and theoretical underpinning of the role of textbooks and the relationship between textbooks and other variables not only in curriculum, teaching and learning but also in a wider educational and social context, which have been largely ignored in previous studies as discussed above (also see Fan 2011). In particular, the existence of textbooks should be viewed from a broader perspective, instead of being treated as an isolated identity.

Secondly, there is a strong need for more confirmatory research about the relationship of the textbook and students' learning outcome.

As reported earlier, the research evidence for a positive correlation between textbooks and students' learning outcome is weak and inconclusive, as it is often based on the comparison of selected textbooks, investigating the differences between textbooks in different countries, and the comparison of students' performance in these countries. In these studies, the issues of whether the selected textbooks are a good representation of all the available textbooks, and whether the students whose academic performances were compared actually used the textbooks analysed, were often ignored or taken for granted.

Thirdly, more research is needed to be directly focused on the issues about the development of textbooks.

As we can see from this survey, virtually all research on mathematics textbooks has been so far focused on the product, that is, the textbook itself. Although some researchers have alluded to textbook design and development issues from different angles (e.g. Macintyre and

Hamilton 2010), there has been a lack of specific research studies centring on the process of development of textbooks, in other words, on how textbooks are produced (see also Fan 2010). The issue raised by Johnsen (1993) about the lack of process-oriented approach in textbook research remains to be tackled.

Fourthly, with a few exceptions, many studies, as reviewed earlier, employed relatively easy and straightforward methods, for example using a small scale (or just a case study), with subjects or participants not being randomly selected, and without control or comparison groups. On the one hand, we must realize that different studies serve different purposes and have different advantages, and hence they should not be ignored. On the other hand, we believe, to address the issue of being "inconclusive" and "trivial", it is necessary for future researchers to go beyond these limitations and employ more advanced and sophisticated methodology in this area of research (see also Fan 2011).

Finally, as we reported above, researchers have only started to look into issues concerning the use and development of electronic textbooks in mathematics and the research is at a very initial stage. There is no doubt that the rapid growth of electronic textbooks in mathematics calls for research in this direction, an area to be emerging and explored.

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